

# Quantum $\text{AdS}^{1+3}$ Black Holes with Effective Cosmological Constant

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**Abstract:** A quantum  $\text{AdS}^{1+3}$  massive and massless black holes with effective cosmological constant induced from non-minimal coupling and supergravities arguments are constructed and discussed in details.

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## 1. Introduction

It was a surprising but interesting result when in 1992-93, Bañados et al. showed that 1+2 “black hole” with negative cosmological constant could be theoretically produced. Such Einstein’s field equations solutions are in fact locally isometric to  $\text{AdS}$  constant curvature background [1,2]. In general, as it was noticed in [3], due to its peculiar asymptotic space with a timelike boundary at spatial infinity, the construction will be possible whatever the dimension of space-time is. Making  $\text{AdS}$  “black hole” in 1+3 dimensions is possible. In this way, S. Aminneborg et al. obtained isometric  $\text{AdS}$  black holes with event horizon that are tori or Riemann surfaces of genus higher than one, with one or two asymptotic regions [3]. In this paper, we will produce “*quantum massless-black hole*” from non-minimal coupling and supergravities arguments, with negative effective *quantum* cosmological constant. By “*quantum*” we mean the presence of the Planck constant “ $\hbar$ ” in our field equations and by “*massless black hole*”, we mean zero  $\text{AdS}$  space with

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zero Schwarzschild mass. But also some physical properties (*in particular temperature and entropy*) of massive (*free-charged and charged*) Schwarzschild-de-Sitter black holes (**SdS**) will be discussed through this paper. Although **AdS** spacetime (*generically arose as ground state in supergravity theory*) does not seem to correspond to the world in which we live, its importance has been noticed in many occasions. In fact, the presence of a negative cosmological constant makes it possible for a black hole to reach stable thermal equilibrium with a heat bath [4].

In fact the idea of massless black holes is not new. They are considered as objects with strange properties. In particular, they are supersymmetric, they saturate all “bounds” and “anti-bounds”, they are annihilated by all supersymmetric and Lorentz charges and they are of Minkowski type being a vacuum state [5,6,7,8,9,10]. In General Relativity (**GR**), when the mass of the Schwarzschild metric goes to zero, the space-time approaches Minkowski’s. However, things are different for the Reissner-Nordström background. It is well-known that in this case, the metric obtained describes a charged massless system with naked singularity at the origin. This singularity could be remedied if for example, we change the sign of the charge was Wick-rotated, described a genuine black hole solutions of the Einstein-anti-Maxwell theory [11]. In addition, despite the successes of Hawking quantum black hole theory, we are left with the feeling that the quantum story is not complete [12,13,14,15,16].

There exist today a lot of interest concerning density perturbations and black hole formation in hybrid two-stage inflations [17,18,19]. It was proved that quantum fluctuations at the time corresponding to the phase transition between the two inflationary stages leads to the formation of a large number of inflating topological defects. It was also showed that density perturbations in hybrid inflation models of the new type can be very large on the scale corresponding to the phase transition [20]. The resulting density inhomogeneities lead to a production of black holes where numbers can be made extremely small, but in general it could be sufficiently large to have cosmological and astrophysical important implications (*for example, the dark matter in the universe*). It is also possible to have two-stage hybrid inflation without suppressed the production of black hole, but where *their typical masses are very small, even massless*. Such models lead to a completely different thermal history of the universe, where post-inflationary reheating occurs via black hole evaporation [20]. The existence of such small-mass or massless black holes represents our first motivation in this work.

Another motivation came from the deepest and important lessons that we have learned over the past modern quantum decade is *that there is no fundamental difference between elementary particles and black holes*. As mentioned a lot of time by ’t Hooft, “*black holes are the natural extension of the elementary particle spectrum*”. This is especially clear in string theory where black holes are simply highly excited string states<sup>2</sup> [21,22,23,24]. Remember that the quantum consistency of string theory requires that extremal black

<sup>2</sup> Zero mass extreme black holes play also a remarkable role in the non-perturbative aspects of string theory; their role in order to understand the conifold singularities that appear in the low energy Lagrangian describing the moduli space of Calabi-Yau vacua of (*type II*) string theory is crucial.

holes be treated as elementary quanta.

An additional motivation came from recent works relating the cosmological constant to the black hole physics. It was known that there is a quantum mechanical tunneling process which, through nucleation of a membrane, induces a transition between two **dS** spaces, lowering the cosmological constant up to zero leaving a black hole behind [25,26,27,28] (*in general, a negative cosmological constant allows the existence of black holes with a topology  $\mathbb{R}^2 \times \Sigma$ , where  $\Sigma$  is a two-dimensional manifold of constant curvature* [29,30,31]).

Finally, remember that in black hole topology, the quasinormal modes, which are typically characterized by a spectrum that is independent of the initial conditions, are a sort of fingerprint of the black hole depending only on its parameters and on the fundamental constants of the system. There exists a lot of interest on the study of these quasinormal modes in asymptotically flat spacetimes with negative cosmological constants, in particular massless black hole with massive scalar field non-minimally coupled to the curvature, with the horizon geometry assumed to have a negative constant curvature [28,32,33,34].

Through the **AdS/CFT** (*C*onformal *F*ield *T*heory) correspondence the quasinormal modes can be related to the relaxation time scale of the associated thermal states<sup>3</sup> [35,36]. Recently, a connection has also been conjectured between the quasinormal modes and critical phenomena of black hole formation in an asymptotically **AdS** background [37,38]. Detailed analysis showed that the energy-momentum flux density of these models vanishes at the asymptotic region when the square of its effective mass is less than zero.

In this work, we explain a new approach describing in particular *static massless quantum black hole* (**SmQBH**) and show that these exotic objects could have important physical features.

## 2. Modified Scalar Curvature From Non-Minimal Coupling

In **GR**, the cosmological constant describes the energy density of the vacuum (*empty space*), and it is a potentially important contributor to the dynamical history of the Universe. In fact, it is not well theoretically understood, what is the physical mechanism which cancels the vacuum energy through the different phase transitions that our universe undergoes! On the other hand, recent observations of indicates that the Universe is in accelerated regime [39,40,41]. If theoretically, the total energy of the universe consists only of ordinary matter and dark matter, than one can interpret the dark energy as the vacuum energy corresponding to the cosmological constant, that is  $\Lambda$  or as the slowly changing energy of a certain scalar field with a vacuum  $\phi$  corresponding to the equation

<sup>3</sup> The interest on **AdS** spacetime was revived by a conjectured duality between string theory in the bulk of **AdS** and conformally **CFT** living on the boundary of **AdS**. The **AdS/CFT** correspondence gives an explicit relation between Yang-Mills theory and string theory. More recently, there has been a renewed interest in **AdS** spacetime since progresses in theories of extra dimensions present us with the enticing possibility to explain some long-standing particle physics problem by geometrical means.

of state  $p = -\rho$  [42]. In both cases the Universe is accelerated with time and approaching de-Sitter (**dS**) regime. From the point of view of string theory, any dimensional parameter must be expressed in terms of the fundamental string scale, and of vacuum expectation values of scalar fields, so that the physics of the cosmological constant is nothing than the physics of the corresponding scalar fields. This complicates the situation, because it is a difficult task to envisage string theory in the context of a non exotic cosmological constant. The corresponding spacetime is the **dS** one having as we know an event horizon (see *S. Weinberg proof of no-go theorem in* [42]).

In fact, the most natural way to talk about a vanishing cosmological constant came from symmetry argument. Supersymmetry (**SUSY**) predicts that the masses of boson and fermions are equal, and this must be broken. But in the context of cosmology, we need the local version, which is supergravity (**SG**). Although **SUSY** may be broken while the cosmological constant remains too small, we lost our rational for a vanishing cosmological constant and undesired fine-tuning reappears again.

In most of the theoretical models of dark energy it is assumed that the cosmological constant is equal to zero and the potential energy of the scalar field driving the present stage of acceleration, slowly decreases and eventually vanishes as the field rolls to  $\phi = \infty$  [43,44,45,46,47]. As a result, after a transient **dS**-like stage, the speed of expansion of the Universe decreases, and the Universe reaches Minkowski regime. Of course, depending on the choice of the model, the flat Universe will become **dS** space, or Minkowski space, or collapse [48,49,50,51]. However, it was found that one can describe dark energy in some extended supergravities that have a **dS** solutions [52,53]. These **dS** solutions correspond in general to the extrema of the effective potentials  $V(\phi)$  for some scalar fields  $\phi$ . An interesting result of these solutions is that the squared mass of these scalars in all theories with  $N = 2$  (*extended supergravities with unstable dS vacua*) is quantized in units of the Hubble constant  $H_o$ . That is  $m^2 = nH_o^2$  where  $n$  are integers of order of unity (*in units of unity Planck Mass*). In extended supergravities with a positive cosmological constant, one always has  $3m^2 = n\Lambda$  where  $\Lambda$  being the cosmological constant. For the  $N = 8$  supergravity, **dS** vacuum corresponds to an unstable maximum  $m^2 = -6H_o^2$  at  $|\phi| \ll 1$  and  $V(\phi) = 3H_o^2(1 - \phi^2)$ . Meanwhile for  $N = 2$  gauged supergravity with stable **dS** vacuum, one has  $m^2 = 6H_o^2$  for one of the scalars and in this case  $V(\phi) = 3H_o^2(1 - \phi^2)$  [54,55,56,57]. Note here that  $m$  is not in these theories, a massive cosmological graviton. In [58,59], it was showed that the graviton has a mass in **AdS** space-time given by  $m_g^2 = -2\Lambda/3$ . The authors were able to show that a perturbation of the Einstein equations, with the presence of the cosmological constant, in a **dS** or **AdS** background, produces exactly the equations obtained from the Fliers formulation for the massive spin-2 field. Their final conclusion is that if the graviton has a real mass not null, than the cosmological constant should be negative.

In application to the cosmological constant problem, this leads to the conclusion that there are ultra light scalars with the mass of the order  $m \simeq H \simeq 10^{-33}eV$ . The significance of this fact and the possibility to use these supergravity models in modern cosmology still have to be well studied and understood. The existence of such ultra

light fields may be a desirable feature for the description of the accelerated universe. Their presence signals that the corresponding potentials are very shallow. In extended supergravity theories ultra light fields necessarily come in a package with too small  $\Lambda$ . Due to the presence of  $\Lambda$ , **SUSY** in **dS** vacua is broken spontaneously, the scale of **SUSY** breaking here is  $10^{-3}eV$ . Before it is coupled to a ‘*visible sector*’, both the tiny  $\Lambda$  as well as the ultra light masses of scalars, that is  $m$ , are protected from large quantum corrections.

Coupling of these theories to real universe is a big problem, of course. If they play a role of a ‘*hidden sector*’, one may ask whether the tiny  $m \simeq H$  will be preserved after coupling to the ‘*visible sector*’. The preservation of the small  $\Lambda$  may imply preservation of small scalar masses  $m$ . Thus, extended supergravities suggest a new perspective for investigation of the cosmological constant problem, intertwined with ultra-light scalars [60,61].

Although the non minimal coupling significantly modifies quantum geometry, the highly non-trivial consistency checks for the emergence of a coherent description of the quantum black hole horizon continue to be met.

In fact, the generalization of the Klein-Gordon equation in curved spacetime includes the possibility of an explicit non-minimal coupling between the scalar field driving inflation and the Ricci curvature of spacetime. Nowadays, there are many reasons to believe that a non-minimal coupling term is present in the Klein-Gordon equation. Quantum corrections generate a non-minimal coupling even if it is absent in the calculation. A nonzero value for this later is also required in order to normalize the theory. Furthermore, it has been argued that a non-minimal coupling term is to be expected whenever the spacetime curvature is large. Due to these considerations, it seems sensible but important to consider an explicit non-minimal coupling between the scalar field  $\phi$  and the scalar curvature  $R$  in the inflationary paradigm.

In a recent paper [61], we introduced, for some scalar field  $\phi$ , a non-minimal coupling between the scalar curvature and the density of the scalar field in the following form  $L = -\xi\sqrt{g}R\phi^*\phi$ ,  $\xi = 1/6$ .  $R$  is the scalar curvature and  $\phi^*$  is the complex conjugate of  $\phi$ . From a view point of quantum field theory in curved space-time, it is natural to consider such a non-minimal coupling. In fact, the case where  $\xi = 1/6$  results in an extension of the property of conformal invariance for massless fields, which is attractive from physical point of view. This parameter describes the strength of the coupling between the curvature of spacetime and the inflaton. Minimal coupling corresponds to  $\xi = 0$ . It was shown that in this case and for a particular scalar negative complex potential field  $V(\phi\phi^*) = 3/4m^2(\omega\phi^2\phi^{*2} - 1)$ ,  $\omega$  being a tiny parameter [62-71], inspired from supergravity inflation theories, ultra-light masses “ $m$ ” are implemented naturally in Einstein field equations (**EFE**), leading to a cosmological constant “ $\Lambda$ ” in accord with observations<sup>4</sup>. In matter-

<sup>4</sup> It has been argued that a non-minimal coupling term-generated by quantum corrections-is to be expected whenever the space-time curvature is large; in most theories that describe inflationary scenarios, it turns out that a value of  $\xi$  different from zero is unavoidable. As a matter of fact, it seems sensible to consider an explicit non-minimal coupling in the supergravities inflationary paradigm.

free background, the scalar curvature was found to be  $R = 4\bar{\Lambda} - 3m^2 = 4\bar{\Lambda}$  where  $\bar{\Lambda} = \Lambda - 3/4m^2$  is the *effective cosmological constant*<sup>5</sup> (in natural units, where “ $\hbar$ ” is the Planck constant and “ $c$ ” being the celerity of light). In this particular case and for  $\omega \ll 1$ , a possible candidate field equations for the scalar curvature  $R$ , will be:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \left(\Lambda - \frac{3}{4}m^2\right)g_{\mu\nu} = 0 \quad (1)$$

We define  $\bar{\Lambda} = \Lambda - 3m^2/4 \equiv \Lambda_1 + \Lambda_2$  or in natural unit  $\bar{\Lambda} = \Lambda_1 - 3m^2c^2/4\hbar^2$  as the effective cosmological constant (one can say that we are dealing here with a theory with two cosmological constant). Here  $\Lambda_1$  (the first) is the standard, that is Einstein’s cosmological constant and  $\Lambda_2$  (the second, which is proportional to the inverse square of the Compton wavelength) is our quantum (so-called due to the presence of “ $\hbar$ ”) second negative cosmological constant. Note that if  $\Lambda_1 = 0$ , the scalar curvature is negative and the spacetime is not Minkowskian as in the standard model. When  $\Lambda_1 = \Lambda_2$ ,  $R = 0$ . In this case,  $m$  is approximately of the same order of the graviton mass described in [58,59] (which is an interesting fact). But the difference is that this our quantum mass is viewed as a boson of spin zero, while the graviton is of spin 2. Remember that the conditions  $\Lambda_1 + \Lambda_2 > 0$  or  $< 0$  play important role in modern cosmological theories, in particular inflation [63-70], in particular when the mass  $m$  is of the order of the Hubble constant.

In what follows, we will be also interested on the case where one of the two cosmological constants, that is, the Einstein cosmological constant, is set equal to zero ( $\Lambda_1 = 0$ ). The resulting equation combines gravity (geometry) with quantum theory ( $\hbar$ ) (another interesting fact). With the presence of the cosmological constant, this equation and in particular our scalar curvature is similar (only in form) to the one obtained by J. Charon long time ago in his complex relativity [72], but of course his vision and motivations were totally different from ours. It is interesting to investigate how much the presence of the quantum ultra light masses (in form of a quantum cosmological constant) our quantum field equations will contribute and change in most of the static standard model, in particular the AdS one.

### 3. AdS<sup>1+3</sup> Spacetime with Negative Effective Quantum Cosmological Constant and Zero Einstein’s Lambda

The requirement of a static field implies as it is well-known that the components of  $g_{\mu\nu}$  of the metric tensor shall not depend upon the time coordinate. The requirement of spherical symmetry can be expressed by writing the metric to be determined in the well-known form [73,74]:

$$ds^2 = g_{00}(r)c^2dt^2 + g_{11}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \quad (2)$$

We work with metric signature  $(+ - - -)$  and we choose naturally  $g_{11}g_{00} = -1$ , with  $g_{00} = 1 + F$  where  $F$  is a spherically symmetric time-independent perturbation. The Ricci tensor and Riemann scalar are found in any standard GR textbook:

<sup>5</sup> In [61],  $8\pi G \equiv \kappa$  was set equal to unity.

$$R_{00} = -\frac{c^2}{2}(1+F) \left[ F'' + 2\frac{F'}{F} \right] \quad (3)$$

$$R_{11} = \frac{1}{2(1+F)} \left[ F'' + 2\frac{F'}{F} \right] \quad (4)$$

$$R_{22} = F + rF' \quad (5)$$

$$R_{33} = \sin^2\theta \left[ rF' + F \right] \quad (6)$$

$$R \equiv g^{\mu\nu} R_{\mu\nu} = - \left[ F'' + 4\frac{F'}{r} + 2\frac{F}{r^2} \right] \quad (7)$$

$$R_{\mu\nu} \neq 0, \mu \neq \nu \quad (8)$$

By choosing  $F = ar^2$ , where  $a$  is a constant, we find using equation (7),  $R = -12a \equiv -3m^2c^2/\hbar^2$ , so that  $a = m^2c^2/4\hbar^2$ . The metric then takes the following form:

$$\begin{aligned} ds^2 &= c^2 \left( 1 - \frac{\Lambda_2 r^2}{3} \right) dt^2 - \frac{dr^2}{\left( 1 - \frac{\Lambda_2 r^2}{3} \right)} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \\ &= c^2 \left( 1 + \frac{m^2c^2}{4\hbar^2} r^2 \right) dt^2 - \frac{dr^2}{\left( 1 + \frac{m^2c^2}{4\hbar^2} r^2 \right)} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \end{aligned} \quad (9)$$

describing a quantum **AdS** with negative effective quantum cosmological constant. Note that in terms of Compton wavelength, the metric (9) can be put in the following form:

$$ds^2 = c^2 \left( 1 + \frac{r^2}{\tilde{\lambda}_C^2} \right) dt^2 - \frac{dr^2}{\left( 1 + \frac{r^2}{\tilde{\lambda}_C^2} \right)} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2) \quad (10)$$

where  $\tilde{\lambda}_C = 2\lambda_C$  represents the **AdS** radius. Note that if  $\Lambda_1 \neq 0$ ,  $g_{00}(r) = 1 - 1/3(\Lambda_1 + \Lambda_2)r^2$ , so that if  $\Lambda_1 > 3m^2c^2/4\hbar^2$ , the metric is of **dS** type, while in case  $\Lambda_1 < 3m^2c^2/4\hbar^2$ , it is of **AdS** type.

For *massive* black hole (*Schwarzschild-de-Sitter spacetime (SdS)*), the potential  $g_{00}(r) = 1 - 2MG/rc^2 - 1/3(\Lambda_1 - 3m^2c^2/4\hbar^2)r^2$  where  $M$  is the Schwarzschild mass. There are two positive roots of the cubic equation  $g_{00}(r) = 0$  corresponding to black horizon  $r_+$  and cosmological horizon  $r_c$ . The third root locates at  $r_- = -(r_+ + r_c)$ . For  $\Lambda_1 > 3m^2c^2/4\hbar^2$ , as the Schwarzschild mass increases, the black hole radius  $r_+$  increases and the cosmological horizon  $r_c$  decreases monotonically. For maximum value of  $M$ ,  $r_+ = r_c = \sqrt{1/\Lambda_1 + \Lambda_2}$ , which is the biggest black hole that can be formed in de Sitter space. Using the results for highly damped quasi-normal modes, one can obtain the area and entropy spectrum of event horizon [75,76,77,78]. Near external **SAdS** black hole, provided  $\Lambda_1 > 3m^2c^2/4\hbar^2$ , the quantum area is found to be of the form  $\Delta A = 24\pi\hbar\sqrt{v_0/\kappa_{BH}^2} - 1/4$  where  $v_0 = \kappa_{BH}^2 l(l+1)$ ,  $\kappa_{BH} \approx (r_c - r_+)/2r_+^2$  is the surface gravity,  $l$  being the angular momentum [79]. This quantum area seems to be not universal for all black holes [80,81,82]. In case  $\Lambda_1 < 3m^2c^2/4\hbar^2$ , we have *Schwarzschild-Anti-de Sitter spacetime (SAdS)*. For large **SAdS** black hole ( $r_0 \ll r_+$  where  $r_0 = \sqrt{3/\Lambda_1 + \Lambda_2}$ ), we find from

$g_{00}(r) = 0$  that the relation between the cosmological constants  $\Lambda_1$  and  $\Lambda_2$  and black hole horizon  $r_+$  is  $\Lambda_1 - 3m^2c^2/4\hbar^2 = -6MG/r_+^2c^2 + 3/r_+^2$ . For a very small black hole, and by considering a system consisting of initially well-separated matter and black hole, the first law of thermodynamics gives approximately the temperature  $T = 1/4\pi\zeta r_c$ ,  $\zeta$  is constant (*providing that  $\Lambda_1 > 3m^2c^2/4\hbar^2$* ). The entropy is given by  $S = \pi r_+^2 = 1/4A$  where  $A$  is the area of the event horizon. This is the same form of entropy  $s$  in asymptotically flat space. Phase transitions of thermal radiation in **SAdS** will be dealt in future work.

Concerning the extreme limit for massive black hole, it has been investigated by several authors [83,84,85,86,87]. An interesting idea is to introduce a non-extremal parameter  $\varepsilon$  than perform the following coordinate change  $r = r_{ext} + \varepsilon r_1$ ,  $t = \varepsilon t_1$  with  $\varepsilon \ll 1$  in the near-extremal limit. One then can show that a large class of black hole solutions admits a well defined limit procedure, in particular the massive **SdS** and **SAdS** spacetime. Following [85], we prove that in the **SdS** case, the final solution is locally  $dS_2 \times S_2$  and is equivalent to the Nariai solution [88], but it's a cosmological solution with  $\Lambda_1 > 3m^2c^2/4\hbar^2$ , while for **SAdS** where  $\Lambda_1 < 3m^2c^2/4\hbar^2$ , the limiting final metric will be  $AdS_2 \times \Sigma_2$ ,  $\Sigma_2$  being a Riemann surface.

Turning now to the *massless* case, the quantum metric (9) has no singularities and its geometry has constant negative curvature  $R = -3\lambda_C^{-2}$  (*assuming the mass  $m$  is constant and  $m^2 > 0$* ). There is no intrinsic horizon of this space-time, from which it follows that the surface gravity yields a divergence. The metric is regular and globally homeomorphic to  $AdS_2 \times S_2$ , or geometrically, this solution is viewed as the direct product of two 2-dimensional manifolds [89,90]. Following [91,92], making massless quantum or microscopic **AdS** black hole seems possible. In fact, formulating quantum entropy of black hole from non-minimal coupling is not new [93,94]. But our approach here is different and indirect. We also note that according to Rindler (*see* [59]) and Deser et al. [58], detectors with constant acceleration " $\hat{a}$ " in **dS** and **AdS** spaces with cosmological constant  $\Lambda$  measure temperatures  $2\pi T = (\Lambda/3 + \hat{a})^{1/2} \equiv \hat{a}_5$ , this later being the *5-acceleration* in the embedding flat 5-space.

For **dS** background, this works correctly, while for **AdS** background, the physics is quite different leading to measure imaginary temperature (*the temperature is well defined down to the critical value  $\hat{a}_5 = 0$ , again in accord with the underlying kinematics*).

The resolution of this paradox is that its motion becomes spacelike. As mentioned in [95], *these considerations were confirmed by explicit calculations of the correlators of a quantum field in AdS, whose result was independent of the different boundary conditions permitted by the AdS causality ambiguities, and led to the standard Unruh Planckian temperature distribution, putting our quantum massless AdS in safe.*

#### 4. Quantum Massless AdS Black Hole and the 3-dimensional Quantum Relativistic Oscillator

In [96], it was shown that a static (1+3) **AdS** metric defines naturally a relativistic harmonic oscillator (**RHO**) in Minkowski space. We follow their arguments and we set

the lagrangian of our metric (18) in the following form:

$$L = -mc \sqrt{1 + \frac{r^2}{\tilde{\lambda}_C^2} - \frac{V^2}{c^2} + \frac{1}{c^2 \tilde{\lambda}_C^2} \frac{(\vec{x} \cdot \vec{V})^2}{1 + \frac{r^2}{\tilde{\lambda}_C^2}}} \quad (11)$$

where  $m$  is the reduced mass of the system and  $V$  being the velocity. The geodesics can be derived from this equation. This Lagrangian defines a 3-dimensional quantum relativistic oscillator in Minkowski space. The Hamiltonian is given by:

$$H^2 = \left(1 + \frac{1}{\tilde{\lambda}_C^2}\right) \left(m^2 c^4 + p^2 c^2 + \frac{c^2}{\tilde{\lambda}_C^2} (\vec{x} \cdot \vec{p})^2\right) \quad (12)$$

$\vec{p}$  being the momentum vector. In fact, the Schrödinger equation associated with this Hamiltonian can be transformed, with a particular normal-ordering prescription, into a Klein-Gordon equation.

In our model, the Klein-Gordon ( $\phi$ ) and Schrödinger wave functions ( $\psi$ ) are related by  $\phi = \sqrt{1 + \frac{r^2}{\tilde{\lambda}_C^2}} \psi$ . The Klein-Gordon equation in our quantum **AdS** background is:

$$\left(\partial_\mu \partial^\mu + \frac{m^2 c^2}{\hbar^2} (1 - 3\xi)\right) \phi = \left(\partial_\mu \partial^\mu + \frac{m_{eff}^2 c^2}{\hbar^2}\right) \phi = 0 \quad (13)$$

with  $m_{eff}^2 = m^2 (1 - 3\xi)$ , where we have used the fact that  $R = -3m^2 c^2 / \hbar^2$ . Here  $\partial_\mu \partial^\mu \equiv \square$  stands for the Laplace-Beltrami operator and  $\xi$  is a numerical factor. Following [38], one can prove that the energy-momentum flux density vanishes at the asymptotic region if  $m_{eff}^2 > 9/4m^2$ . The solution of equation (13) is done in [38]. It was shown that for an **AdS** background  $\phi$  must be restricted to be a purely ingoing wave at the horizon, a reason why the energy-momentum flux density should vanish at the asymptotic region<sup>6</sup>.

In our case, the Laplace-Beltrami operator is given by:

$$\square = \frac{1}{\left(1 + \frac{r^2}{\tilde{\lambda}_C^2}\right)} \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - 2 \frac{r}{\tilde{\lambda}_C^2} \frac{\partial}{\partial r} - \left(1 + \frac{r^2}{\tilde{\lambda}_C^2}\right) \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r}\right) + \frac{\tilde{L}^2}{r^2} \quad (14)$$

where

$$\tilde{L}^2 = -\frac{1}{\sin\theta} \frac{\partial}{\partial \theta} \left(\sin\theta \frac{\partial}{\partial \theta}\right) - \frac{1}{\sin^2\theta} \frac{\partial^2}{\partial \varphi^2} \quad (15)$$

is the orbital angular momentum operator. After solving the Klein-Gordon equation, one finds the following wave functions (*for the detailed of the calculations, the reader is referred to [96]*):

$$\phi(\vec{x}, t) = e^{-i\tilde{c}t} Y_m^l(\theta, \varphi) \left(1 + \frac{r^2}{\tilde{\lambda}_C^2}\right)^{-p/2} r^l \phi_l^p(r) \quad (16)$$

<sup>6</sup> Topological **AdS** black hole with negative constant curvature exist also for spacetime dimensions greater than 4, that is  $d > 4$  and also for gravity theories containing higher order of the curvature (*see [38] and references therein*).

where  $\tilde{c} = \tilde{\lambda}_C \omega$ ,  $\omega$  being the angular velocity and  $p$  is an arbitrary positive parameter. The functions  $\phi_l^p(r)$  verify the following hypergeometric differential equation:

$$\left\{ (1 - \rho) \rho \frac{d^2}{d\rho^2} + \left( l + \frac{3}{2} - \left( l + \frac{5}{2} - p \right) \rho \right) \frac{d}{d\rho} - \frac{1}{4} (p(p - 2l - 3) + l(l + 3) - N^2 + 12\xi) \right\} \phi_l^p(\rho) = 0 \quad (17)$$

where  $\rho = -r^2/\tilde{\lambda}_C^2$  and  $N = mc\tilde{\lambda}_C/\hbar$ . With the regularity condition at the origin and the square integrability of the wave functions, the energy spectrum is then given by:

$$\begin{aligned} E &= \left( \frac{3}{2} + 2n + l + \frac{1}{2} \sqrt{9 + 4 \left( \frac{m^2 c^2}{\hbar^2} \right) \tilde{\lambda}_C^2 - 48\xi} \right) \hbar\omega \\ &= \left( \frac{3}{2} + 2n + l + \frac{1}{2} \sqrt{9 + 4 \left( \frac{\tilde{\lambda}_C^2}{\lambda_C^2} \right) \tilde{\lambda}_C^2 - 48\xi} \right) \hbar\omega \\ &= \left( \frac{3}{2} + 2n + l + \frac{1}{2} \sqrt{25 - 48\xi} \right) \hbar\omega \\ &= \left( \frac{3}{2} + 2n + l \right) \hbar\omega + \left( \frac{1}{2} \sqrt{25 - 48\xi} \right) \hbar\omega \end{aligned} \quad (18)$$

where we have used the fact that  $\tilde{\lambda}_C = 2\lambda_C$ . For the ground state energy, we have:

$$E = \frac{3}{2} \hbar\omega + \left( \frac{1}{2} \sqrt{25 - 48\xi} \right) \hbar\omega \quad (19)$$

This quantum **AdS** black hole behaves as a relativistic harmonic oscillator. The system is exactly solvable.

Observe that when  $\xi = 25/48$ , the second term of equation (19) disappears, leaving the ordinary energy spectrum of 3-dimensional non-relativistic oscillator, while in [96], this is obtained only in the non-relativistic limit, that is when . For this value of , the Klein-Gordon became:

$$\left( \partial_\mu \partial^\mu + \frac{\tilde{m}^2 c^2}{\hbar^2} \right) \phi = 0 \quad (20)$$

where  $\tilde{m}^2 = -27/48m^2 < 0$  (*superluminal or sub-eV particles: tachyons*). For this value of  $\xi$ , the quantum theory is not well defined, if not at this level, not well understood. While for  $\xi = 1/3$ ,  $\square\phi = 0$  (*massless Klein-Gordon scalar wave equation*) and  $E = 3\hbar\omega$  for its corresponding ground states with angular velocity three times the standard one, that is  $\omega' = 3\omega$ .

In conclusion, the energy spectrum coincides, up to the ground state energy, with that of the non-relativistic oscillator, but the presence of the mixing parameter could have, as we have seen, important impacts on the whole problem.

## 5. Charged Quantum AdS Black Holes, Tachyons and the Validity of the Quantum Theory

Due to its huge interesting physical aspects, much work has gone into deep understanding of charged **AdS** black holes (*thermodynamics, holography, quantum fluctuations...*), in particular, its quantum gravity features<sup>7</sup> [12,97,98,99,100,101]. This motivates us to study the impact of our theory on this later. When an electromagnetic fields is added to our field equations (1) as:

$$(T^{\mu\nu})_{e.m.} = \varepsilon_0 \left( F_{\alpha}^{\mu} F^{\alpha\nu} + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta} \right) \quad (21)$$

where  $F^{\alpha\beta}$  is the tensor of the electromagnetic field and  $\varepsilon_0$  is the permittivity constant, we may found a metric in the centrosymmetric form of equation (2) with:

$$g_{00}(r) = 1 - \frac{2MG}{rc^2} + \frac{q^2 G}{4\pi\varepsilon_0 c^4 r^2} - \frac{1}{3} \left( \Lambda_1 - \frac{3m^2 c^2}{4\hbar^2} \right) r^2 \quad (22)$$

$$\equiv 1 - \frac{2\tilde{M}}{r} + \frac{\tilde{Q}^2}{r^2} - \frac{1}{3} \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) r^2 \quad (23)$$

$$\equiv 1 - \frac{r_+}{r} - \frac{1}{3} \frac{r_+^3}{r} \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) + \frac{\tilde{Q}^2}{r^2} - \frac{\tilde{Q}^2}{r_+ r} - \frac{1}{3} \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) r^2 \quad (24)$$

where for simplicity, we set  $\tilde{M} = MG/c^2$ ,  $\tilde{Q}^2 = q^2 G/4\pi\varepsilon_0 c^4$  and  $\tilde{m} = mc/\hbar$ . is the Schwarzschild gravitational mass and  $G$  is the gravitational constant. When  $\Lambda_1 < 3m^2 c^2/4\hbar^2$ , the metric corresponds to the Reissner-Nordström black hole with a negative effective cosmological constant. The mass of the corresponding black hole is given by:

$$M = \frac{1}{2} \left( r_+ - \frac{1}{3} \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) r_+^3 + \frac{\tilde{Q}^2}{r_+} \right) \quad (25)$$

and satisfies the laws of black hole thermodynamics with a temperature:

$$T_{BH} = \frac{1 - \frac{\tilde{Q}^2}{r_+^2} - \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) r_+^2}{4\pi r_+} \quad (26)$$

and a potential  $\phi = Q/r_+$ . In general  $r_+$  and  $Q$  are independent, but in the extremal case, they get related as:

$$1 - \frac{\tilde{Q}^2}{r_+^2} - \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) r_+^2 = 0 \quad (27)$$

<sup>7</sup> The whole focus of the physics of charged black hole is to find physical mechanisms or a suitable quantum mechanics that resolve the Hawking paradox. It has seemed plausible to many black hole physicists that extremal charged black holes (**ECBH**) are the endpoint of Hawking evaporation if the black hole manages to retain its charge, that is, stabilize its charge. This particular and strange situation was ignored by most high-energy researchers. Because the formation and evaporation of **ECBH** can be studied within the framework of a controlled approximation scheme, we hope in the future to extract from them important information or novel physics in the aim to better understand quantum neutral black holes.

Note that when  $\Lambda_1 = 3/4\tilde{m}^2$  and  $r_+ = \tilde{Q}$ ,  $T_{BH} = 0$ . When dealing with cosmological Einstein-Maxwell theory, equation (22) takes the form:

$$g_{00}(r) = 1 - \frac{2\tilde{M}}{r} + \frac{\tilde{Z}^2}{r^2} - \frac{1}{3} \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) r^2 \quad (28)$$

where  $\tilde{Z}^2 = \tilde{Q}^2 + \tilde{H}^2$  being the magnetic charge [102,103]. From the behavior of the curvature invariants:

$$R^2 = 16 (\Lambda_1 + \Lambda_2)^2 \quad (29)$$

$$R_{mn}R^{mn} = 4 \left( \frac{\tilde{Z}^4}{r^8} + (\Lambda_1 + \Lambda_2)^2 \right) \quad (30)$$

$$C_{mnpq}C^{mnpq} = \frac{48}{r^4} \left( \frac{\tilde{M}}{r} - \frac{\tilde{Z}^2}{r^2} \right)^2 \quad (31)$$

one can remark that such solutions possess a single physical singularity located at  $r = 0$ . Here indices  $m, n, \dots$  are “curved” world indices [104]. Note that when  $\Lambda_1 = -\Lambda_2 = 3/4\tilde{m}^2$ ,  $R = 0$ . Also if the classical Einstein’s cosmological constant is set equal to zero, keeping the other one, the curvature  $R \neq 0$ . In asymptotically flat space-time, that is  $\Lambda_1 = -\Lambda_2 = 3/4\tilde{m}^2$ , for  $\tilde{Z}^2 < \tilde{M}^2$ , there exist two horizons at radii  $\tilde{\rho}_{\pm} = \tilde{M} \pm \sqrt{\tilde{M}^2 - \tilde{Z}^2}$ . While for  $\Lambda_1 \neq -\Lambda_2$ , the condition  $g_{00}(r) = 0$  is a quartic algebraic equation for  $r$ , that is:

$$-\frac{1}{3} (\Lambda_1 + \Lambda_2) r^4 + r^2 - 2\tilde{M}r + \tilde{Z}^2 = 0 \quad (32)$$

One may follow [104] and show that for  $\Lambda_1 > 3/4\tilde{m}^2$ , the existence of an additional event horizon  $\approx \sqrt{3/\Lambda_1 + \Lambda_2}$  constituting the “outer edge” of the **dS** universe in the given coordinate system. The Hawking effective temperature for the Reissner-Nordström solutions is given by:

$$T^{effective} = \frac{\kappa}{2\pi} = \frac{1}{4\pi\tilde{\rho}} \left| 1 - \frac{\tilde{Z}^2}{\tilde{\rho}^2} - \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) \tilde{\rho}^2 \right| \quad (33)$$

where  $\kappa$  is the surface gravity at the horizon [12]. Taking  $\tilde{Z}^2 = \Lambda_1 = 0$  gives the temperature of an uncharged cosmological **AdS** black hole. It is evident from equation (33) that horizons with zero Hawking temperature are located at simultaneous roots  $\tilde{\rho}$  of both  $g_{00}$  and  $g'_{00}$  (at double roots of  $g_{00}(\tilde{\rho})$ ). When such a double root exists,  $g_{00}(r)$  takes the following form:

$$g_{00}(r) = \left( 1 - \frac{\tilde{\rho}}{r} \right) \left( 1 - \frac{1}{3} \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) (r^2 + 2\tilde{\rho}r + 3\tilde{\rho}^2) \right) \quad (34)$$

with the corresponding critical relationships

$$\tilde{M} = \tilde{\rho} \left( 1 - \frac{2}{3} \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) \tilde{\rho}^2 \right) \quad (35)$$

$$\tilde{Z}^2 = \tilde{\rho}^2 \left( 1 - \left( \Lambda_1 - \frac{3}{4}\tilde{m}^2 \right) \tilde{\rho}^2 \right) \quad (36)$$

For  $\Lambda_1 \leq 3/4\tilde{m}^2$  (*supersymmetric black holes*), all positive values of  $\tilde{\rho}$ ,  $\tilde{M}$  and  $\tilde{Z}^2$  are admitted. While for  $\Lambda_1 > 3/4\tilde{m}^2$ , the only allowed maximum radius is  $\tilde{\rho}_{\max} = 1/\sqrt{\Lambda_1 - 3/4\tilde{m}^2}$  at which the charge vanishes. For  $0 < \tilde{\rho} < \tilde{\rho}_{\max}$ , there is an extra positive root given by

$$\sigma = \sqrt{\frac{3}{\Lambda_1 - \frac{3}{4}\tilde{m}^2} - 2\tilde{\rho}^2 - \tilde{\rho}} \quad (37)$$

Note that when  $0 < \tilde{\rho}^2 < 1/2(\Lambda_1 - 3/4\tilde{m}^2)^{-1}$ , the extra horizon at  $\sigma$  is outside the cold region at  $\tilde{\rho}$ , while for  $1/2(\Lambda_1 - 3/4\tilde{m}^2)^{-1} < \tilde{\rho}^2 < (\Lambda_1 - 3/4\tilde{m}^2)^{-1}$ , it is inside. Note that the effective temperature at  $\sigma$  is given by [104]:

$$T_{\sigma}^{\text{effective}} = \frac{\sigma}{2\pi(\sigma^2 + 2\tilde{\rho}\sigma + 3\tilde{\rho}^2)} \left( 1 - \frac{\tilde{\rho}}{\sigma} \right) \left( 1 - \frac{\tilde{\rho}^2}{\sigma^2} \right) \quad (38)$$

When  $\tilde{\rho} = 0$ , one can easily show that the black hole disappears and find  $T_{dS}^{\text{effective}} = 1/2\pi\sqrt{1/3(\Lambda_1 - 3/4\tilde{m}^2)}$  corresponding to a background of effective thermal background in a pure **dS** cosmology [105] with  $\Lambda_1 > 3/4\tilde{m}^2$ . Note that in this case,  $\Lambda_1 = 0$  is not permitted unless  $\tilde{m}^2 < 0$ , or in other words, we admit the possible existence of tachyons (*we will discuss this critical possibility at the end of this paragraph*). Finally, note that in case  $\Lambda_1 = -\Lambda_2 = 3/4\tilde{m}^2$ ,  $T_{dS}^{\text{effective}} = 0$ .

Before passing to the *massless* case, note that for *massive* black hole and for a certain range values of the Reissner-Nordström charge and Schwarzschild mass, the spacetime has three Killing horizons: inner and outer black hole horizons and a positive cosmological constant, that is  $\Lambda_1 > 3m^2c^2/4\hbar^2$ . The external limit, in which the inner and outer black hole horizons become coincident, occurs when the Schwarzschild mass is less or equal to the absolute value of the Reissner-Nordström charge ( $\tilde{M} \leq |\tilde{Q}|$ ), with equality in the case  $\Lambda_1 = \Lambda_2$ , while in the standard case, it corresponds to zero cosmological constant<sup>8</sup>. Following [106], one can show that spacetimes containing a number of charge equal to mass black holes with  $\Lambda_1 \geq \Lambda_2$ , have supercovariantly constant spinors, suggesting that they may be minimum energy states in a positive energy construction<sup>9</sup>.

In what follows, the Einstein cosmological constant  $\Lambda_1$  as well as the Schwarzschild gravitational mass are set equal to zero, that is,  $\Lambda_1 = M = 0$ . We left with a quantum metric of the following form:

$$ds^2 = c^2 \left( 1 + \frac{r^2}{\tilde{\lambda}_C^2} + \frac{r_0^2}{r^2} \right) dt^2 - \frac{dr^2}{\left( 1 + \frac{r^2}{\tilde{\lambda}_C^2} + \frac{r_0^2}{r^2} \right)} - r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

<sup>8</sup> A nonzero cosmological constant  $\Lambda = -3g^2$  arises in gauged  $N = 2$  supergravity, where  $g$  is the coupling constant of the gravitino with the  $U(1)$  gauge field. If  $g$  is real, then  $\Lambda < 0$ .

<sup>9</sup> Particle production and positive energy theorems for charged **dS** black holes with positive effective cosmological constant will be left to a future work.

$$= c^2 \left( \frac{r^4 + r^2 \tilde{\lambda}_C^2 + r_0^2 \tilde{\lambda}_C^2}{r^2 \tilde{\lambda}_C^2} \right) dt^2 - \frac{dr^2}{\left( \frac{r^4 + r^2 \tilde{\lambda}_C^2 + r_0^2 \tilde{\lambda}_C^2}{r^2 \tilde{\lambda}_C^2} \right)} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (39)$$

$$\xrightarrow{m \rightarrow 0} c^2 \left( 1 + \frac{r_0^2}{r^2} \right) dt^2 - \frac{dr^2}{\left( 1 + \frac{r_0^2}{r^2} \right)} - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \quad (40)$$

which is the classical charged massless black hole described in [11]. Here  $r_0^2 = q^2 G / 4\pi \epsilon_0 c^4$ . Naturally, such a metric could be obtained starting from *Einstein-Maxwell-Anti-de-Sitter (EMAdS)* action<sup>10</sup> [100].

In fact, the metric (40) describes a charged massless quantum system and has two naked singularities. Note that  $r^4 + r^2 \tilde{\lambda}_C^2 + r_0^2 \tilde{\lambda}_C^2$  is a quartic algebraic equation for  $r$  and has four roots:

$$r_{\pm}^2 = -\tilde{\lambda}_C^2 \pm \sqrt{\tilde{\lambda}_C^4 - 4r_0^2 \tilde{\lambda}_C^2 / 2} \quad (41)$$

$$= -2 \left( \lambda_C^2 \pm \sqrt{\lambda_C^2 (\lambda_C^2 - r_0^2)} \right) \quad (42)$$

It is evident that both roots are squared negative. One may ask what happens if  $r^2$  is not allowed to be negative. In this case,  $r^2$  will become greater than zero only if we suppose that, from *hypothetical* point of view, we are dealing with superluminal or sub-eV particles (*for example, hypothetical neutrinos with negative square masses*)<sup>11</sup>, than the horizons described in equation (41) or (42) could play significant role. Maybe it will be of interest if black hole evaporation will be responsible of the production of tachyons in the visible universe<sup>12</sup>.

To study this critical hypothetical situation, where  $r^2 > 0$  (at least one of  $r_{\pm}$ ), we suppose that  $m^2 < 0$ , that is  $\lambda_C^2 < 0$ . If to a certain order, we suppose that  $r_0^2 = -\eta \lambda_C^2$  where  $\eta$  is a positive parameter, then  $r_{\pm}^2 = -2\lambda_C^2 [1 \pm \sqrt{1 + \eta}]$ , and only  $r_-^2 > 0$  adding another singular horizons to the problem other than the origin. In this special case,  $m^2 = -\eta (4\pi \epsilon_0 \hbar^2 c^2 / q^2 G) = -\eta M^2 < 0$  where  $M = \alpha_{FS}^{-1/2} m_{Pl}$  (*a massive particle*), with  $\alpha_{FS} = q^2 / 4\pi \epsilon_0 \hbar c$  is the fine structure constant and  $m_{Pl} = \sqrt{\hbar c / G}$  is the Planck constant. Note that in **GUT's** (*Grand Unification Theories*) the unification mass is given by

<sup>10</sup> By scaling the gauge field  $A_{\mu}$ , in order to absorb the prefactors involving the  $U(1)$  gauge coupling into the action.

<sup>11</sup> On the experimental discovery that the mass-square of neutrino is negative, the author is referred to [107]. In [108], based on this discovery, a quantum theory for superluminal neutrino is proposed. But, if neutrino is a tachyon, then all the weak interaction in which neutrino participates need to be restudied. See also [109].

<sup>12</sup> We believe that if tachyons are emitted by a black hole, the quantum theory will not be suitable in explaining the physics of black hole, or at least must be defined in another way in this work; whatever is the case, we will not deal with this problem in this work, neither dealing with the possibility of evaporation of black holes into tachyons, we are mentioning this case from phenomenological point of view, not only.

$m_{GUT} \approx \alpha_{FS}^{1/2} m_{Pl}$ , characterizing gravito-electroweak unification scale [110,111]. This seems interesting to have in function of the **GUT** mass scale,  $m_{GUT} \approx \alpha_{FS}^{-1} m_{Pl}$ . In addition to the gravitons (*as well as the other particles*) are radiated with the Hawking temperature from the black hole [112,113], the implementation of our quantum mass into this theory is an important feature and could have important impacts on black hole and massive remnants. Some quantum gravity researchers claim that the Hawking process terminates with the production of a large, massive remnant which carries all of the information missing from the outgoing radiation. Whether the remnant subsequently decays or whether it is long lived, the information stored in it becomes accessible [114,115,116]. Electroweak gauge bosons have masses of the order of  $10^2 GeV (/c^2)$  while masses of additional bosons involved in gravito-electroweak unification are expected to be still higher. These are at least eleven orders of magnitude higher than sub-eV range indications for neutrino masses. If under these circumstances we suspect that the sub-eV particles are created in a spacetime where gravitational effects of massive gauge bosons may become important, than one may ask on the nature of the spacetime group around a gravito-electroweak vertex [117]. In this later reference, the authors modeled this spacetime as **dS** background and find that sub-eV particles may carry a negative mass square of the order of  $-3(3/8\pi^2)(M_{GUT}^4/m_{Pl}^2)$ .

## 6. Conclusions

The study of extreme and nearly extreme black holes, in particular the massless **BTZ** black hole background as well as supersymmetric massless black holes has recently been increased, mainly due to the link with one of the *quantum* puzzle which is still unsolved: the issue related to the statistical physical interpretation of the Bekenstein-Hawking entropy. Explaining and understand physically this point is recognized as an essential hallmark to complete the quantum theory of gravity.

In this work, we have used the non-minimal coupling to implement the quantum Compton wavelength naturally in the Einstein field equations. At this level, the field equations are viewed with two cosmological constant or an effective one  $\Lambda = \Lambda_1 + \Lambda_2$  where  $\Lambda_1$  is the standard, that is Einstein's cosmological constant and  $\Lambda_2$  (*which is proportional to the inverse square of the Compton wavelength*) is our quantum second negative cosmological constant. We have been also interested on the topology where  $\Lambda_1 = 0$ . We have then constructed a quantum massless **AdS** black hole singular free, regular and globally homeomorphic to  $AdS_2 \times S_2$ .

For *massive SdS* black hole, there are two radius corresponding to black horizon ( $r_+$ ) and cosmological horizon ( $r_c$ ). For  $\Lambda_1 > 3m^2c^2/4\hbar^2$ , as the Schwarzschild mass increases, the black hole radius  $r_+$  increases and the cosmological horizon  $r_c$  decreases monotonically. For maximum value of  $M$ ,  $r_+ = r_c = \sqrt{1/\Lambda_1 + \Lambda_2}$ , which is the biggest black hole that can be formed in de Sitter space. In case  $\Lambda_1 < 3m^2c^2/4\hbar^2$ , we have **SAdS**. For a very small black hole, and by considering a system consisting of initially well-separated matter and black hole, the first law of thermodynamics gives approximately

the temperature  $T = 1/4\pi\zeta r_c$  providing that  $\Lambda_1 > 3m^2c^2/4\hbar^2$ . The entropy is given by  $S = \pi r_+^2 = 1/4A$  where  $A$  is the area of the event horizon. This is the same form of entropy  $s$  in asymptotically flat space.

We showed also that quantum **AdS** free-charged black hole behaves as a relativistic harmonic oscillator. While for charged black holes, interesting features arise. First, for zero classical Einstein's cosmological constant ( $\Lambda_1 = 0$ ) and negative *quantum* one ( $\Lambda_2 = -3/4\tilde{m}^2$ ), the square of the scalar curvature  $R^2 > 0$ . For  $\Lambda_1 > 3/4\tilde{m}^2$ , there exist an additional event horizon  $\approx \sqrt{3/\Lambda_1 + \Lambda_2}$  constituting the “*outer edge*” of the **dS** universe in the given coordinate system. In asymptotically flat space-time, in particular for  $\tilde{Z}^2 < \tilde{M}^2$ , there exists two horizons at radii  $\tilde{\rho}_\pm = \tilde{M} \pm \sqrt{\tilde{M}^2 - \tilde{Z}^2}$ . Taking  $\tilde{Z}^2 = \Lambda_1 = 0$ , the Hawking temperature corresponds to an uncharged cosmological **AdS** black hole, while in the standard case, it corresponds to a **dS** one. Horizons with zero Hawking temperature are found to be located at double roots of  $g_{00}(\tilde{\rho})$ . When such a double roots exists, and in particular for  $\Lambda_1 > 3/4\tilde{m}^2$ , the only allowed maximum radius is  $\tilde{\rho}_{\max} = 1/\sqrt{\Lambda_1 - 3/4\tilde{m}^2}$  at which the charge vanishes. When  $\tilde{\rho} = 0$ , one can easily show that the black hole disappears with an effective **dS** temperature given by  $T_{dS}^{effective} = 1/2\pi\sqrt{1/3(\Lambda_1 - 3/4\tilde{m}^2)}$  corresponding to a background of effective thermal background in a pure **dS** cosmology.  $\Lambda_1 = 0$  in this case is not permitted unless we admit the possible existence of tachyons, that is  $\tilde{m}^2 < 0$ . Phase transitions of thermal radiation in **SAdS** will be dealt in future work. Another important question we would like to answer in another work is how the presence of the two cosmological constants  $\Lambda_1$  and  $\Lambda_2$  reflects itself in properties of the motion of test particles and photons, the photon escape cones and the embedding diagrams [118].

For *massless* black hole, an interesting *phenomenological* feature arises when we admit the presence of other than  $r = 0$  horizon. In this case, we are argued to accept the possible production of sub-eV particles, that is, tachyons with negative mass square of the same order of the square of gravito-electroweak unification scale masses, pushing us to ask about the validity of the quantum theory and whether or not this later is really suitable to explain the mystery of quantum gravity black holes physics, and about the nature of physics that could describe their internal structures. Thus, a more careful study of their backgrounds, as well as their holographic properties will be crucial in order to shed light on their real quantum nature.

One certainly needs understand better all these issues (*in addition to string-black holes interactions, foams, holographic properties,...*) and we hope to return to them in further publications.

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