

Non Linear Assessment of Musical Consonance

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Abstract: The position of intervals and the degree of musical consonance can be objectively explained by temporal series formed by mixing two pure sounds covering an octave. This result is achieved by means of Recurrence Quantification Analysis (RQA) without considering neither overtones nor physiological hypotheses. The obtained prediction of a consonance can be considered a novel solution to Galileo’s conjecture on the nature of consonance. It constitutes an objective link between musical performance and listeners’ hearing activity.

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1. Introduction

According to a current definition, *consonance* is the “intervallic relationships in sound frequencies producing sounds of repose” [1], and a basic problem in the field of musical acoustics concerns the fact that since musical consonance is a graduated criterion, in principle it should be possible: i) to produce an order of merit (ranking) among intervals, and ii) to find an algorithmic derivation of consonance ranking [2].

An excellent example of the consonance-dissonance issue is due to Galileo Galilei [3],

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who imagined two identical pendula, starting to move together. If both of them have the same period, corresponding to the unison interval, the simplest possible event occurs: the two pendula oscillate in perfect synchrony. If the period of one of the pendula is doubled, configuring an octave interval, a momentary event of perfect synchrony will occur periodically. Changing the ratio between the two periods from 2/1 to 3/2, 4/5 or 11/12, the recurrence of the synchronization event becomes less frequent in time, and the inter-period length changes in a complex, nonlinear fashion giving rise to less consonant combinations. This is known as Galileo’s “simple-frequency ratio” theory.

Galileo’s idea is qualitative and metaphoric. However, by using a rather simple nonlinear algorithm, it is possible to show that Galileo’s formulation is surprisingly accurate, being able to catch the prominent features of a standard psychological ranking of musical consonance.

Nonlinear mechanisms have been recently found at the root of the cochlear response in mammals and a correct coupling between non-linearities has been proposed [4-5-6].

In this Letter the recurrence (non-linear) approach is used in a “key experiment”: acoustic sample containing all possible intervals between two pure sounds in the span of an octave were generated and analyzed. We show that the recurrence structure of these samples is strongly related to the musician’s use of the word consonance (psychoacoustic consonance).

2. Methods

2.1 Sound generation.

A pair of pure sinusoidal tones are generated by independent channels and then mixed together by means of the sound editor Cool-Edit Pro (Sintrillium Software Corporation, Phoenix, AZ), see Fig.1(a). The first tone is a constant pure tone at fixed frequency f_1 and the second tone is a *glissando* tone, with a variable, monotonically growing, frequency, f_2 . The sum of the two tones accounts for all the intervals in the span of the octave. The signal was generated, with $f_1=264$ Hz (C4) as root frequency and f_2 going from 200 to 600 Hz, with both f_1 and f_2 being pure sinusoidal waveforms. The corresponding signal was recorded with a sampling rate of 6000 Hz and lasted four seconds (24000 points). Furthermore, each point of the resulting sample can be labeled with an interval ratio, that is, the ratio between the changing frequency and the constant one. In table I the ratios that characterize significant signal points, and the specific name in the scale of “just intonation”, are indicated [7]. A constant interval ratio is obtained mixing two pure tones at fixed f_1 and f_2 frequencies.

2.2 Estimation recurrences

Recurrence Quantification Analysis (RQA) is a relatively new nonlinear technique originally developed by Eckmann et al [8] as a purely graphical method and then made

quantitative by Webber and Zbilut [9]. The technique was successfully applied to a number of different fields ranging from physiology [10] to molecular dynamics [11]. Recently, we exploited RQA also for its ability to provide a synthetic description of the otoacoustic emissions [12].

The notion of recurrence is simple: for any ordered time series, a recurrence is a value which repeats itself within an assigned tolerance (= Radius). Thus, given a reference point, X_0 , and a sphere of radius r centered on it, a point X is said to recur if

$$B_r(X_0) = \{X : \|X - X_0\| \leq r\} \quad (1)$$

The application of this computation produces a Recurrence Plot (RP) which, according to the Webber and Zbilut algorithm [9], is obtained from the initial measured signal by means of the following steps:

- an *embedding matrix* of dimension ‘d’ is built, where the first column is the time series of the signal and the following d-1 columns are time-lagged (according to a “lag” parameter) copies of it;

- a *distance matrix*, where an element in the i,j position corresponds to the Euclidean distance between the i_{th} and j_{th} rows of the embedding matrix, is derived.

Thus, the Recurrence Plot is simply a graphical representation of the distance matrix, namely a square array where each element is represented as a black dot if the corresponding element in the distance matrix is lower than a fixed cut-off value (see Fig.1 (b-c-d)).

In this work the RQA working parameters are: *Embedding Dimension* $m=8$, *Radius* = 20% of the mean distance value (two epochs are considered as recurrent if their euclidean distance is below the 20% of average distance between all the epoch pairs) and *Line* = 35 (scoring of at least 35 consecutive recurrent points is needed to consider a diagonal line as deterministic).

Webber and Zbilut [9] developed several strategies to quantify the features of the recurrence plots originally pointed out by Eckmann et al. [8]. Recurrence analysis was performed using the appropriate subprogram of the RQA suite, called Recurrence Quantification Epochs (RQE), in which the recurrence variables are computed within a moving window (*epoch*) shifted by a given number of points (delay) throughout the whole sample. This implies the setting of two other working parameters namely *window length* = 500 and *windows shifting* = 10. The RQE subprogram was used for the glissando samples providing, for each window, a set of RQA variables calculated on the basis of the number and disposition of dots in the recurrence plot. Such variables were:

- Percent Recurrence (% REC), the fraction of the plot occupied by recurrent points, i.e. by epoch pairs whose distance is lower than a threshold (Radius). This is a measure of the recurrent (both periodic and auto-similar) features of the signal.
- Percent Determinism (%DET), the fraction of recurrent points aligned into upward diagonal segments (deterministic lines). This indicates the degree of deterministic structuring due to the presence of “quasi-attractors”, i.e. portions of the phase space in which the system lies for a longer time than expected by chance alone.
- Entropy (ENT), a Shannon entropy estimated over the length distribution of deter-

ministic lines and linked to the richness of deterministic structuring.

- MAXLINE, the maximum number of points in diagonal lines.

It is actually possible to define other descriptors in RQA [9]; however, we checked that the Principal Component Analysis (PCA) carried out on %REC, %DET, ENT and 1/MAXLINE, produces the same result as if performed on the whole set of descriptors. For this reason, only the four above mentioned RQA descriptors were dimensionally reduced by agency of PCA [13].

RQA was computed by means of a public domain suite of programs [14-15], and PCA by the appropriate subroutines of the SASTM statistical package.

2.3 Results and Discussion

One prominent feature of a recurrence plot [15] is related to the typical representation of the recurrences picked up in a signal (see for example Fig. 1(b)). Fig. 1(c) and 1(d) show recurrence plots of the consonant *perfect fifth* and dissonant *diminished fifth*, respectively (for the interval name see Table I). This corresponds to a preliminary, still qualitative, proof of Galileo’s conjecture: the consonant pairing gives rise to the most regular (simple) recurrence pattern.

A windowing procedure (RQE) carried out on the acoustic sample shown in Fig.1(a), provides for each window the set of recurrence variables to be submitted to PCA. The first component (PC1), explaining 68% of the total variance, was plotted in Fig. 2 vs. the interval ratio. The good agreement between the peaks and the position of most of the musically relevant intervals is noticeable. Moreover, the area of the peaks is proportional to the accepted order of consonance of the musical intervals, with a square correlation coefficient (R-square) equal to 0.86 (see Fig. 3). This correlation proves the feasibility of the reconstruction of a perceptive consonance rank, like the empirical Malmberg’s order of merit [16], by means of some mathematical treatment of the acoustic signal, thus opening the way for a general paradigm about the nature of consonance.

The consonance dimension of music is an objective link between musical performance and listeners’ hearing activity. Thus, it is possible to consider the results depicted in figure 2 as a quantitative assessment of the consonance profile of musical fragments.

Moreover, the link between consonance and the recurrent structures of the superposition of two pure tones can be taken as an empirical evidence of Galileo’s conjecture based on the consideration of the purely mechanical model provided by two oscillating pendula.

Another conclusion of our work is that to deal with the relative psychoacoustic merit of different sounds, consideration of *overtones* and, in general, of complex harmonics, is not required. For the same reason, even the timbral characteristics of sound are not necessary to predict musical consonance.

These results were tested for several different frequencies; however, it is worth noting that in the bass range (C2 frequency-65.406 Hz) the “simple-frequency ratio” theory does not hold for pure tones. In some instances, in fact, more complex ratios are more consonant than simpler ones. This behavior is actually not managed by our extension

of Galileo's conjecture, and can be the result of the lack of sensitivity of the auditory system in a frequency range quite far from the speaking frequencies, and hence difficult to estimate by a purely psychoacoustical scale.

All in all, it may be safely stated that the recurrence approach is both intuitive and powerful as a paradigm to understand consonance.

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Legend to Table and Figures

Table I. General features of Musical Intervals.

The table reports the synthetic name (key) of “musically relevant” intervals, their name, the two frequency ratio (“just intonation” scale) and a psychoacoustic scale of consonance (Malmberg’s order of merit)[18].

Figure 1. Sound Sample and recurrence plots.

(a) and (b) panels show the first 7500 points of a sound sample used in this work and the corresponding recurrence plot, respectively; in the lower left corner of panel (b), notice the framed critical bandwidth centered at “unison”. The sound sample was generated, with $f_1=264$ Hz (C4) as root frequency and f_2 going from 200 to 600 Hz, both f_1 and f_2 being pure sinusoidal waveforms. The signal was recorded with a sample rate of 6000 Hz and lasted four seconds (24000 points).

Panels (c) and (d) refer to a consonant (perfect fifth) and a dissonant (diminished fifth) interval, respectively, showing four beats along the diagonal (corresponding to a plot of 2000×2000 points). Notice the much higher regularity of the beat shapes in (c) as compared to (d). The working parameters used in the generation of the recurrence plot are explained in the text.

Figure 2. First principal component (PC1) scores against interval ratios.

The scores of the first principal component (PC1) extracted from the RQE variables on sound sample of Fig.1(a) are reported as a function of the interval ratio. The most significant peaks are labeled with the interval ratio names in the scale of just intonation (see Table I).

Figure 3. PC1 scores and consonance ranking.

The peaks’ areas in Figure 3, corresponding to PC1 scores vs. interval ratio, are reported vs. Malmberg’s interval order of merit [18]. Notice the good linear relation as indicated by the R-square ($R_{sq} = 0.86$).

Table I

KEY	Name	Interval Ratio	Malmberg's order of merit
U	unison	1:1	-
ST	semitone	16:15	1
MaT	major tone	9:8	2.5
Mi3	minor third	6:5	5.5
Ma3	major third	5:4	7.2
P4	perfect fourth	4:3	7.2
Tt	diminished fifth (tritone)	64:45	4.2
P5	perfect fifth	3:2	9.8
Mi6	minor sixth	8:5	6.5
Ma6	major sixth	5:3	8
HMi7	harmonic minor seventh	7:4	-
Mi7	minor seventh	9:5	3.5
Ma7	major seventh	15:8	2.2
P8	octave	2:1	11

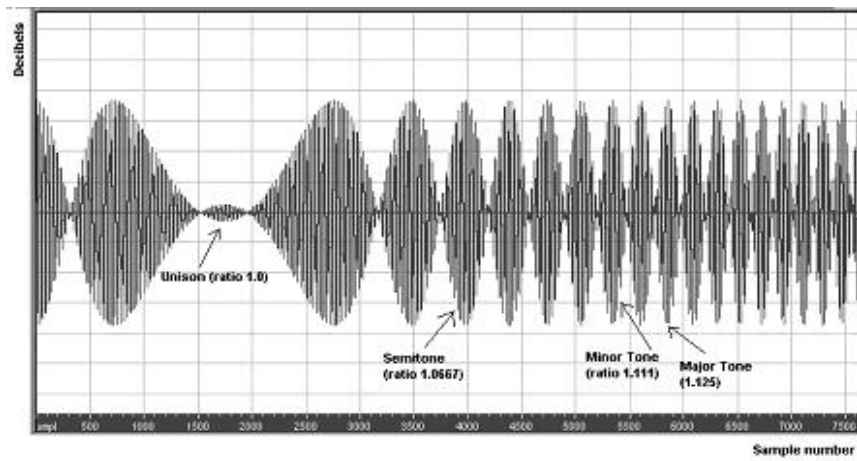
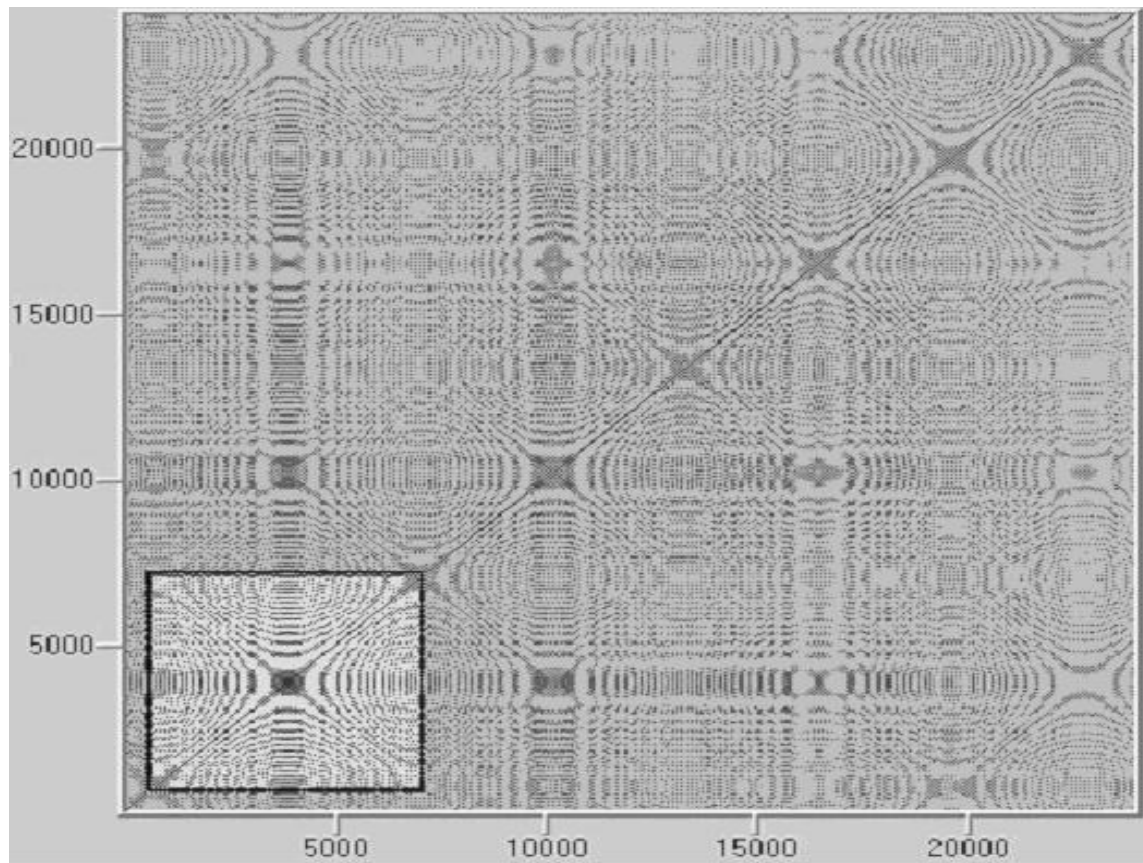
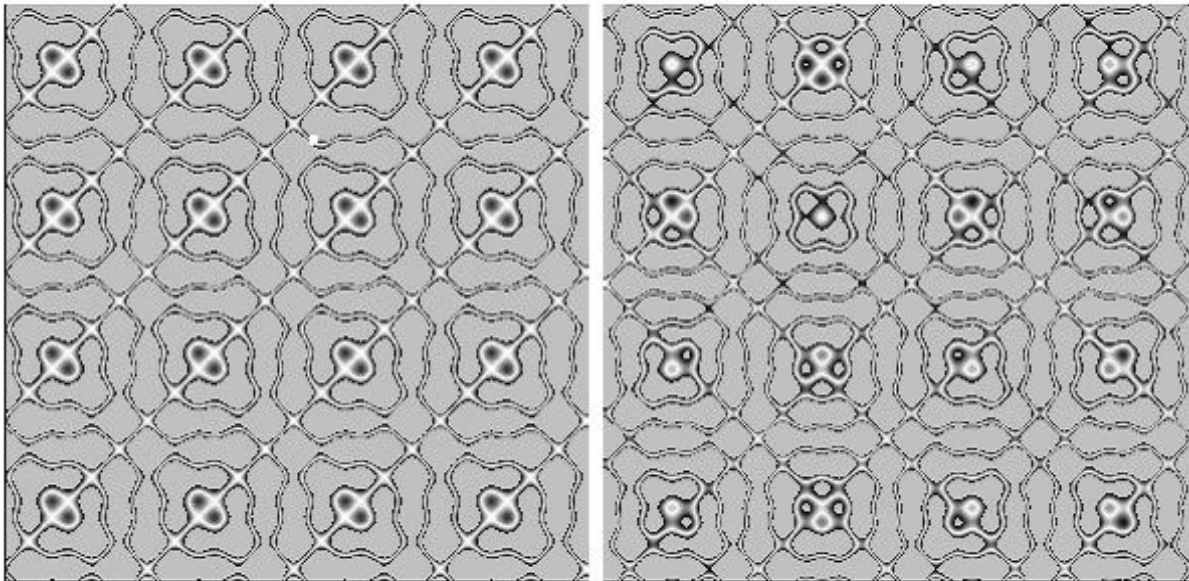


Figure 1 (a) Sound sample's waveform (7500 points)



(b) Full view of the recurrence plot (24000*24000 points)



(c) consonant interval ($3/2$)

(d) dissonant interval ($64/45$)

**(c)-(d) Detailed view of the recurrence plot for different interval ratios
(2000*2000 points)**

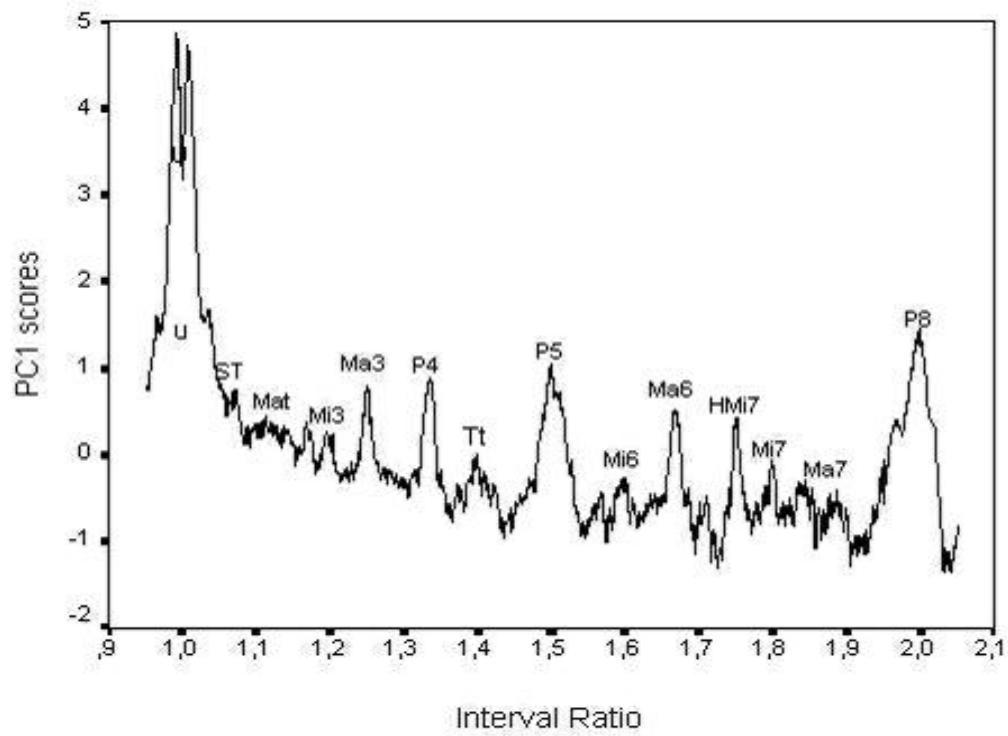


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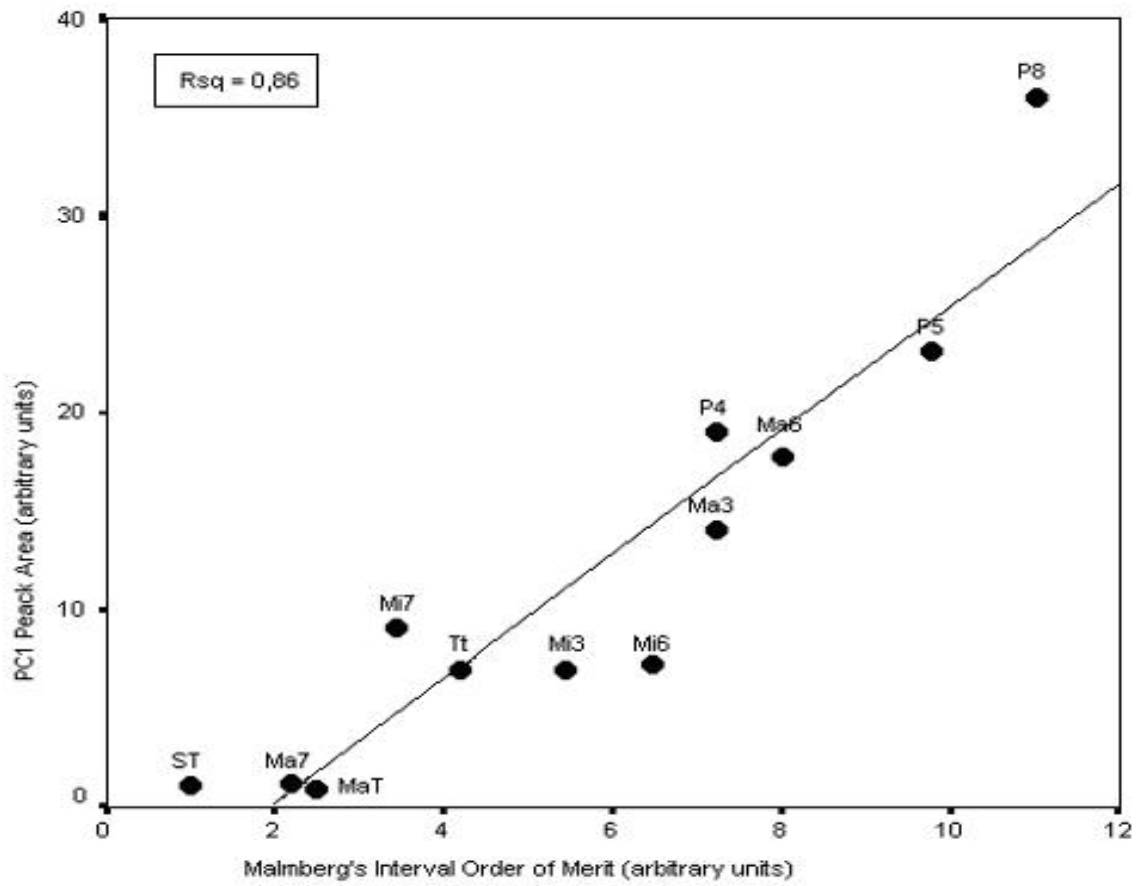


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