Hidden Symmetry, Excitonic Transitions and Two-Dimensional Kane’s Exciton in the Quantum Well

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Abstract: The influence of hidden symmetry on two-dimensional excitonic states in semiconductor quantum wells is investigated. It is shown that excitonic states in quantum wells, with the parabolic dispersion law for the electron and hole, and Sommerfeld’s coefficients for excitonic transitions are determined only with the principle quantum number within the framework of two-dimensional Coulomb potential. This is a result of hidden symmetry of two-dimensional Coulomb problem, conditioned by the existence of two-dimensional analog of the Runge-Lentz vector. For the narrow gap semiconductor quantum well with the non-parabolic dispersion law of electron and hole, in the two-band Kane model, it is shown that two-dimensional excitonic states are described in the frames of analog of the Klein-Gordon equation with the two-dimensional Coulomb potential. Non-stability of the ground state of the two-dimensional Kane’s exciton investigated.

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1. Introduction

Due to the existence of the size quantization effects in the semiconductor nanostuctures it became possible to realize such a low dimensional systems, which initially had only model character. One of such is the two-dimensional Coulomb system, which is obtained during the impurity and excitonic states formation in quantum wells and superlattices [1-8]. Mathematically two-dimensional Coulomb potential is defined by the
following expression [9]:

\[ V(x, y) = -\frac{Ze^2}{\sqrt{x^2 + y^2}}, \]

where \( Z \) is the charge number. Corresponding Schrödinger equation in the Cartesian coordinates is the following

\[ \frac{-\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Psi - \frac{Ze^2}{\sqrt{x^2 + y^2}} \Psi = E \Psi. \]

It should be mentioned that two-dimensional hydrogen atom, being a system with the hidden symmetry, can be investigated simultaneously in several coordinate systems as: polar [10], parabolic [11] and elliptical [12]. In the absence of fields, it is more appropriate to use polar coordinate system.

As it was mentioned, one of the causes for the formation of two-dimensional Coulomb systems in semiconductor quantum wells is the formation of excitons in them during the interband optical absorption [13]. Since the system under is two-dimensional the exciton in it is also two-dimensional (provided strong size quantization [14]) and the energy of electron-hole interaction can be described in the frames of two-dimensional Coulomb potential (1) and charge number \( Z = 1 \). Thus, two-dimensional excitonic states will also have the hidden symmetry, natural for the two-dimensional Coulomb problem. It is clear that this fact should be reflected on the character of the excitonic transitions in quantum well.

On the other hand it should be mentioned that not all semiconductor quantum wells have the parabolic dispersion law of the charge carriers. There are compounds in which, due to the existence of strong interaction of valence band with the band of conductivity, non parabolic dispersion law for the electrons and holes can exist [15]. For that case, theoretical description of the two-dimensional excitons should be done with the consideration of the kinetic operator, which deviates from standard quadratic. It is natural to expect that the above mentioned case can bring to the disappearance of the occasional degeneration of the two-dimensional Coulomb problem discussed.

In this theoretical work the influence of the hidden symmetry on the excitonic transitions in semiconductor quantum wells with the standard dispersion law for the electrons and holes, as well as peculiarities of the two-dimensional excitonic states in narrowband semiconductor quantum wells with the consideration of non-parabolicity of the charge carriers’ dispersion law are investigated.

As was mentioned above we are studying the two-dimensional electron-hole system with the interaction potential (1). Corresponding Schrödinger two-particle equation will have the following form.

\[ \frac{-\hbar^2}{2m_e} \Delta_e + \frac{\hbar^2}{2m_h} \Delta_h + \frac{e^2}{|\vec{\rho}_e - \vec{\rho}_h|} \Psi = E \Psi, \]

where \(-\frac{\hbar^2}{2m_{e(h)}} \Delta_{e(h)}\) is the operator of kinetic energy of the electron (hole). Introducing reduced mass

\[ \mu = \frac{m_em_h}{m_e + m_h}, \]
relative coordinate
\[ \vec{\rho} = \vec{\rho}_e - \vec{\rho}_h, \]
and applying standard procedure for the transition from two- to one-particle problem we will obtain two-dimensional Schrödinger equation
\[ - \left\{ \frac{\hbar^2}{2\mu} \Delta + \frac{e^2}{\rho} \right\} \Psi = E \Psi. \tag{4} \]
In polar coordinates equation (4) will get to the following form [9]
\[ - \frac{\hbar^2}{2\mu} \left\{ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + \frac{1}{\rho^2} \frac{\partial^2}{\partial \varphi^2} \right\} \Psi_{n,m} (\rho, \varphi) - \frac{e^2}{\rho} \Psi_{n,m} (\rho, \varphi) = E \Psi_{n,m} (\rho, \varphi). \tag{5} \]
The well-known solution for this equation is following
\[ \Psi_{n,m} (\rho, \varphi) = e^{im\varphi} \frac{1}{\sqrt{2\pi}} R_{n,m} (\rho), \tag{6} \]
where
\[ R_{n,m} (\rho) = \left[ \frac{2 (n - |m|)!}{a_{ex}^2 (n + \frac{1}{2})^3 \{n + |m|\}!} \right]^\frac{1}{2} \left( \frac{\rho}{a_{ex} \lambda} \right)^{|m|} e^{\left( \frac{-e^2}{2a_{ex} \lambda} \right)} \frac{L_{n+|m|}^{|m|}}{L_{n+|m|}^{|m|}} \left( \frac{\rho}{a_{ex} \lambda} \right), \tag{7} \]
and where \( a_{ex} = \frac{\hbar^2}{\mu e^2} \) is the exciton radius, \( \lambda = \sqrt{- \frac{Ry^*}{4E}} \), \( Ry^* \) is the effective Rydberg’s energy, \( m \) is the magnetic quantum number, \( n = n_\rho + |m| \) is the principle quantum number \( (m = 0, \pm1, \pm2, \ldots, \pm n) \), \( n_\rho \) is the radial quantum number, \( L_n^m (x) \) is the Laguerre’s polynomial. Corresponding energetic spectrum will be
\[ E_{ex}^n = - \frac{\mu e^4}{2\hbar^2 (n + \frac{1}{2})^2}. \tag{8} \]
As it can be seen from (8), two-dimensional excitonic states are degenerated with the degeneracy order
\[ g (n) = 2n + 1. \tag{9} \]
In the paper [16] it is shown, that this degeneration is connected with the existence of the two-dimensional analog of the Runge-Lentz vector in the two-dimensional Coulomb problem. Components of that vector are
\[ \hat{\gamma}_1 = \frac{1}{\sqrt{-2E}} \left( \frac{\hat{z}}{\rho} - \hat{P}_y \hat{L}_z + i \frac{\hat{P}_z}{\rho} \right), \tag{10} \]
\[ \hat{\gamma}_2 = \frac{1}{\sqrt{-2E}} \left( \frac{\hat{z}}{\rho} + \hat{L}_z \hat{P}_x - i \frac{\hat{P}_y}{\rho} \right). \]
These operators along with plane rotation generator \( \hat{\gamma}_3 = \hat{L}_z = -i\hbar \frac{\partial}{\partial \varphi} \) Abelian group \( O(2) \) define hidden group symmetry \( SO (3) \) commuting with the Hamiltonian of the two-dimensional Coulomb problem and providing corresponding degeneracy order. At the same time generators of this group satisfy the commutation relation
\[ [\hat{\gamma}_i, \hat{\gamma}_j] = i\varepsilon_{ijk} \hat{\gamma}_k, \tag{11} \]
where \( \varepsilon_{ijk} \) is the absolute anti-symmetric tensor.
2. Excitonic Transitions in Quantum Wells

During the investigation of the optical characteristics of the semiconductors it was revealed that under the certain conditions of the light absorption in semiconductors may be obtained such a conditions in which electron transiting from valence band to the conductivity band can create a bound state with the hole in the valence band, without reaching the conductivity band [14]. In other words excitonic state occurs and corresponding transitions are called excitonic. In the theory of semiconductor optical properties it is shown that the intensity of the excitonic transitions is characterized with the Sommerfeld coefficients [14], which are equal to the values of the squared modulus of excitonic wave function in $\rho = 0$
\[ Z_1 (n) = |\Psi_{n,m} (0)|^2 , \]
if $\Psi_{n,m} (0) \neq 0$, and
\[ Z_2 (n) = |\nabla \Psi_{n,m} (0)|^2 , \]
if $\Psi_{n,m} (0) = 0$. For the first case transitions are called allowed, and for the second case forbidden.

Let’s define the Sommerfeld coefficients for the excitonic transitions in the quantum well, when exciton is described in the frames of above mentioned two-dimensional Coloumbic problem. We should use the wave function (7). From that it is obvious that allowed transitions take place only for the states for which $m = 0$. Thus the corresponding Sommerfeld’s coefficient will have the following form [14]
\[ Z_1 (n) = |\Psi_{n,0} (0)|^2 = \frac{1}{\pi a_{ex}^2 (n + \frac{1}{2})^3} . \]

For the forbidden transitions case, first, we should write Laguerre’s polynomial in the waive function in the (7) explicit form
\[ L_{n+|m|}^{2|m|} (x) = \sum_{k=0}^{n-|m|} \frac{(-1)^{k+2|m|} (n + |m|)!^2 x^k}{(n - |m| - k)! (2 |m| + k)! k!} . \]

Direct calculations show that $\nabla \Psi_{n,m} (0)$ is nonzero when $m = 0$ and $m = \pm 1$. So for the Sommerfeld’s coefficient we can take
\[ Z_2 (n) = \sum_{i=-1}^{1} |\nabla \Psi_{n,i} (0)|^2 . \]
Calculating the corresponding values for the gradients we can see that
\[ |\nabla \Psi_{n,0} (0)|^2 = \frac{4 (n + \frac{1}{2})^2}{\pi a_{ex}^2 (n + \frac{1}{2})^5} . \]
\[ |\nabla \Psi_{n,\pm 1} (0)|^2 = \frac{n (n + 1)}{\pi a_{ex}^4 (n + \frac{1}{2})^5} . \]
Taking into account (16-18) for the Sommerfeld’s coefficient in the case of forbidden transitions we get:

\[ Z_2(n) = \frac{3(n + \frac{1}{2})^2 - \frac{1}{4}}{2\pi a_\text{ex}^4 (n + \frac{1}{2})^3}. \]  

(19)

As it follows the intensity of the forbidden excitonic transitions in the two-dimensional case depends only on the principle quantum number \( n \). For its fixed values, due to the existence of hidden symmetry of the discussed problem, for the states with different \( m \) and \( n_\rho \), but with the same \( n = n_\rho + |m| \) we get the same intensity for the excitonic transitions \( Z_2(n) \). In Table 1. Sommerfeld’s coefficients for different forbidden transitions are brought. To sum up, we can conclude that for the strong quantization in quantum

\begin{table}[h]
\begin{tabular}{|c|c|c|}
\hline
\( n = 1 \) & \( n = 2 \) & \( n = 3 \) \\
\hline
\( n_\rho = 1, m = 0 \) & \( n_\rho = 2, m = 0 \) & \( n_\rho = 3, m = 0 \) \\
\hline
\( n_\rho = 0, m = 1 \) & \( n_\rho = 1, m = 1 \) & \( n_\rho = 2, m = 1 \) \\
\hline
\( n_\rho = 0, m = -1 \) & \( n_\rho = 1, m = -1 \) & \( n_\rho = 2, m = -1 \) \\
\hline
\( Z_2(1) = \frac{1}{\pi a_\text{ex}^4} \frac{104}{243} \) & \( Z_2(2) = \frac{1}{\pi a_\text{ex}^4} \frac{296}{3125} \) & \( Z_2(3) = \frac{1}{\pi a_\text{ex}^4} \frac{584}{16807} \) \\
\hline
\end{tabular}
\end{table}

Table 1. Sommerfeld’s coefficients for the forbidden two-dimensional excitonic transitions for different values of main quantum number \( n \).

well, when we can use two-dimensional exciton model, due to the hidden symmetry of the two-dimensional Coulomb problem, Sommerfeld’s coefficients of the exciton transitions are expressed only with principle quantum number. This makes possible for the case of forbidden transitions to realize the same intensity of the two-dimensional excitonic transitions for different states. As soon as we assume the quazi two-dimensionality of the exciton and insert the \( z \) coordinate into Hamiltonian the hidden symmetry of the problem disappears and Sommerfeld’s coefficients now depend also on \( m \) and \( n_\rho \).

3. Two-dimensional Kane’s Exciton

Along with the consideration of quazi two-dimensionality, also consideration of non-parabolicity of dispersion law for the electron and hole in the narrow band semiconductor quantum wells in two-dimensional Coulomb problem can bring to the disappearance of the hidden symmetry. What is important, that electron-hole interaction potential can be described within the frames of two-dimensional potential (1). In Kane’s works it was shown that consideration of interband interactions in semiconductor brings to the deviation of the electron-whole dispersion law from the quadratic form [15]. In two-band approximation, when the effective masses of electron and whole are equal, Kane’s dispersion law becomes analogous to the relativistic one, however, naturally, there is nothing relativistic, simple mathematical coincidence. If we introduce the band interaction parameter \( s(s \approx 10^8 \text{ sm/s}) \), which is defined through interband dipole matrix element, the
corresponding dispersion law will take the following form [15]:

\[ E = \sqrt{p^2 s^2 + \mu^2 s^4}. \]  

(20)

Thus, within the frames of two-band Kane’s approximation for defining the excitonic states one does have to solve corresponding steady-state Klein-Gordon equation with the Coulomb potential. It is clear, that for the two-dimensional excitonic states in narrowband semiconductor quantum well problem reduces to the investigation of the two-dimensional Klein-Gordon equation with the Coulombic interaction term (1).

In the polar coordinates Klein-Gordon equation for the Coulomb field takes the following form [17]

\[
\left[ \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) + 1 \frac{\partial^2}{\partial \varphi^2} \right] \Psi_{n,\rho,m} + \left[ \frac{2Ze^2}{\hbar^2 c^2 \rho} + \frac{Z^2 e^4}{\hbar^2 c^2 \rho^2} - \frac{1}{\hbar^2 c^2} \left( \mu^2 c^4 - E^2 \right) \right] \Psi_{n,\rho,m} = 0,
\]

(21)

where \( \mu \) is the electron mass.

Representing electron’s wave function as a compound of radial \( R(\vec{\rho}) \) and angular \( \Phi(\varphi) \) functions \( \Psi_{n,\rho,m}(\vec{\rho}, \varphi) = R_{n,\rho,m}(\vec{\rho}) \Phi_m(\varphi) \) and using the variable separation method, from (1) we obtain two equations which define \( R(\vec{\rho}) \) and \( \Phi(\varphi) \):

\[
\rho^2 \frac{d^2 R_{n,\rho,m}}{d\rho^2} + \rho \frac{dR_{n,\rho,m}}{d\rho} + \left[ \frac{2Ze^2 E}{\hbar^2 c^2 \rho} + \frac{Z^2 e^4}{\hbar^2 c^2} - \left( \mu^2 c^4 - E^2 \right) \rho^2 - m^2 \right] R_{n,\rho,m} = 0 \quad (22)
\]

and

\[
\frac{d^2 \Phi_m}{d\varphi^2} + m^2 \Phi_m = 0, \quad (23)
\]

where \( m = 0; \pm 1; \pm 2; \ldots \) is the magnetic quantum number, \( n_\rho \)—is the radial quantum number.

From the (23) for the normalized angular functions \( \Phi_m(\varphi) \) we get

\[ \Phi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi}. \]  

(24)

For the radial equation (22) first, let’s introduce following notations:

\[ \varepsilon = \frac{1}{\hbar c} \left( \mu^2 c^4 - E^2 \right)^{1/2}, \quad \lambda = Z\alpha E \left( \mu^2 c^4 - E^2 \right), \quad \alpha = \frac{e^2}{\hbar c}, \]  

(25)

And dimensionless variable \( r = 2\varepsilon \rho \). Then for the \( R(r) \) we come to the following equation

\[
R''_{n,\rho,m} + \frac{R'_{n,\rho,m}}{r} + \left( \frac{\lambda}{r} + \frac{Z^2 \alpha^2 - m^2}{r^2} - \frac{1}{4} \right) R_{n,\rho,m} = 0. \quad (26)
\]

With the consideration of the solutions of (26) in two limiting cases \( r \to 0 \) and \( r \to \infty \), we will seek general solution in the following form.

\[ R_{n,\rho,m}(r) = r^S \exp \left(-r/2\right) u_{n,\rho,m}(r), \]  

(27)

where \( S = \sqrt{m^2 - Z^2 \alpha^2}. \)
Putting (27) in (26) for \( u(r) \) we will get Kummer’s equation [18]:

\[
ru' + (2S + 1 + r) u' + \left( \lambda - S - \frac{1}{2} \right) u = 0. \tag{28}
\]

The solution of the (28), which satisfies standard conditions, can be expressed through the confluent hypergeometrical function \( 1F1(\alpha, \beta; x) \). Thus for the radial wave function we can take

\[
R_{n,0} (r) = C_{n\rho,0} r^S \exp \left( -r/2 \right) \frac{1}{\Gamma(1+S)} \left( \frac{r}{2} \right)^{1/2}.
\]

(29)

where \( C_{n\rho,0} \) is the normalization constant.

Energetic spectrum is determined from the breaking of the degenerated hypergeometrical series \( 1F1(\alpha, \beta; x) \) condition and has the following form

\[
E_{n\rho,m} = \mu c^2 \left[ \frac{1 - Z^2 \alpha^2}{Z^2 + \left( n_{\rho} + \frac{1}{2} \sqrt{|m|^2 - Z^2 \alpha^2} \right)^2} \right]^{1/2}.
\]

(30)

Let’s discuss the energetic spectrum and the system behavior when \( m = 0 \) in more detail. As it follows from the (30) in that case energy becomes complex. Latter is connected with the occurrence of the instability in the problem [19]. The cause of this instability is easy to understand, if consider, that the term \( Z^2 e^4/2\mu c^2 \rho^2 \) in the equation (21) in the non-relativistic theory can be regarded as a part of potential energy, which is attractive. When \( m = 0 \) this attraction becomes prevailed, the fall on to center, which means that system becomes unstable. This fall could be seen from the wave function when \( r \to 0 \).

Indeed, when \( m = 0 \) \( S \) becomes imaginary quantity \( S = iZ\alpha \) and \( R(r) \sim e^{iZ\alpha ln r} \). When \( r \to 0 \) wave function oscillates infinite number of times [19].

Let’s remember, that similar situation occurs when solving the three-dimensional relativistic hydrogen-like atom problem [20], but here the difference is that this instability takes place when \( Z\alpha > \frac{1}{2} \).

When \( Z\alpha << 1 \) from (30) for the atom’s energy we have

\[
E_{n\rho,m} = \mu c^2 \left[ \frac{1 - Z^2 \alpha^2}{2 \left( n + \frac{1}{2} \right)^2} - \frac{Z^4 \alpha^4}{2 \left| m \right| \left( n + \frac{1}{2} \right)^3} \left( 1 - \frac{3}{4} \left| m \right| \left( n + \frac{1}{2} \right) \right) \right],
\]

(31)

where \( n = n_{\rho} + |m| \). As it follows from (31) consideration of the relativity brings to the disappearance of the degeneration by \( m \) and now the group of symmetry of the corresponding Hamiltonian is \( O(2) \). In other words, consideration of the non-parabolicity of the dispersion law of charge carriers brings to the disappearance of the hidden symmetry in the Coulomb problem. Second term in the decomposition is the energy of the non-relativistic two-dimensional atom. As it is seen from (31) relativistic corrections increase bond energy.

As it was mentioned, Kane’s dispersion law of the charge carriers in the two-band approximation has the form analogous to the relativistic one. That is why we can two-particle Hamiltonian function of the exciton problem reduces to the effective one-particle one and apply the results obtained above to the Kane’s exciton.
For the Kane’s exciton case the role of the light speed plays the parameters, mass of the free electron $\mu$ replaces with the effective mass $\mu_e$ of the electron in the crystal. From that for the effective constant of the fine structure $\alpha^*$ we can take

$$\alpha^* = \frac{e^2}{\hbar s}. \quad (32)$$

In the two-band interaction effective masses of the electron and hole are equal, $\mu_e = \mu_h$, which makes it possible to bring the two-particle Hamilton function to the one-particle. Indeed, passing to the new coordinate system, in which $\mu_e \overrightarrow{\rho}_e + \mu_h \overrightarrow{\rho}_h = 0$, and introducing $\overrightarrow{\rho} = \overrightarrow{\rho}_e - \overrightarrow{\rho}_h$, as well as considering that $\overrightarrow{\rho}_e = \frac{\mu_h}{\mu_e + \mu_h} \overrightarrow{\rho}$ and $\overrightarrow{\rho}_h = -\frac{\mu_e}{\mu_e + \mu_h} \overrightarrow{\rho}$, for the electron and hole impulse operators we will get

$$\hat{P}_e = \frac{\mu_e + \mu_h}{\mu_h} \hat{\overrightarrow{P}}, \quad \hat{P}_h = -\frac{\mu_e + \mu_h}{\mu_e} \hat{\overrightarrow{P}}, \quad (33)$$

where $\hat{\overrightarrow{P}} = -i\hbar \frac{\partial}{\partial \overrightarrow{\rho}}$. Switching to the new coordinate $\overrightarrow{\rho}' = \frac{\overrightarrow{\rho}}{4}$ and introducing effective mass and charge, $\mu' \equiv 2\mu$, $e' \equiv 2e$, we come to the following equation

$$(\hat{P}'^2 s^2 + \mu'^2 s^4)^2 \Psi_{n_{\rho},m} = \left( E_{ex}^{n_{\rho},m} + \frac{e'^2}{\rho^2} \right)^2 \Psi_{n_{\rho},m} \quad (34)$$

which fully coincides with the Klein-Gordon equation for the two-dimensional hydrogen-like atom, where instead of $\mu$ we have $\mu'$, $e - e'$, $c - s$, and $\hat{P}' = 2\hat{\overrightarrow{P}}$. This allows us to apply to the Kane’s exciton main results, obtained during the solution of the two-dimensional relativistic hydrogen atom which are:

1. Exciton’s energetic spectrum has the form

$$E_{ex}^{n_{\rho},m} = \left[ 1 - \frac{4\alpha'^2}{4\alpha'^2 + \left(n_{\rho} + \frac{1}{2} + \sqrt{m^2 - 4\alpha'^2}\right)^2} \right] \mu'^2 s^2, \quad (35)$$

1. The states of the system with $m = 0$ are unstable.

2. Degeneration of energy levels, which takes place in the case of two-dimensional exciton with the standard dispersion law, disappears.

And finally after the standard procedure of the normalizations of the radial part of the exciton’s wave function

$$\int_0^\infty R_{n_{\rho},m}^2 (\rho) \rho d\rho = 1, \quad (36)$$

with the consideration of [18]

$$I = \int_0^\infty e^{-\chi x} x^{\nu - 1} F^2 \left( -n; \gamma; \chi x \right) dx = \frac{\Gamma(\nu)n!}{\chi^\gamma(\gamma + 1)\cdots(\gamma + n - 1)} \times$$

$$\times \left\{ 1 + \sum_{p=0}^{n-1} \frac{n(n-1)\cdots(n-p)(\gamma - \nu - p - 1)\cdots(\gamma + p)}{(p+1)!\gamma(\gamma + 1)\cdots(\gamma + p)} \right\}, \quad (37)$$
for $\Psi_{n,\rho,m}(\vec{\rho}, \varphi) = R_{n,\rho,m}(\vec{\rho}) \Phi_m(\varphi)$ eventually we can get

$$\Psi_{n,\rho,m}(\vec{\rho}, \varphi) = \frac{1}{\alpha_{ex}} \sqrt{\frac{2(2S+1) \cdots (2S+n_\rho)}{\pi \Gamma(2S+2)n_\rho!}} \left(1 + \frac{2n_\rho}{2S + 1}\right) e^{im\varphi} e^{-i\vec{\rho} \cdot F(-n_\rho; 2S + 1; 2\vec{\rho})}.$$

(38)

**Conclusion**

Due to the hidden symmetry in two-dimensional Coulomb system, the Sommerfeld coefficients of the forbidden excitonic transitions depend only on the principal quantum number. As a result of this for different magnetic and radial quantum numbers of the excitonic states can correspond the same Sommerfield’s coefficient. Situation radically changes if we consider narrowband quantum well. In that case due to the existence of interband interaction the dispersion law of the electron and hole becomes non-parabolic and Hamiltonian of the two-dimensional exciton system no longer have the hidden symmetry. In particular, if we investigate the model of two-dimensional exciton within the frames of two-band Kane’s approximation, the states with the magnetic quantum number $m = 0$ do not realize at all, and other states are no longer degenerated by $m$.

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