

# Self-Interacting Scalar Field and Galactic Dark Halos

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**Abstract:** We construct dark halo models which are supported by a self-interacting scalar field. The possibility that the energy density of such a field which could produce dark matter and dark energy inside and outside of the galactic dark halos is explored.

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## 1. Introduction

According to Newton's laws, the rotation curve of spiral galaxies should decrease as  $\frac{1}{\sqrt{|r|}}$  with increasing distance from center in regions where the mass density almost vanishes. But the galaxies rotation curves are quite flat up to large distances from the center. According to various arguments, the mass of the visually luminous matter is not sufficient to bind clusters gravitationally, and significant amounts of non-luminous dark matter should exist as well.

The existence of dark matter in different scales of the universe, ranging from galaxies to cluster of galaxies up to cosmological scales have been asserted by astronomers. In 1933 Zwicky was the first who measured the radial velocity of eight galaxies of the Coma cluster and he deduced the total mass of clusters should be more than the amount of visible mass [1]. In the 1970's majority of astronomers were convinced that similar situation exists for the spherical halos around individual galaxies [2, 3, 4].

Measurements of the rotation curves of high red-shift galaxies [5] and of gravitational lensing of light by clusters [6] show that dark matter lies within and particularly around

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galaxies, and not spread between them.

To date, various possibilities have been presented about baryonic dark matter. MA-CHO's (Massive Compact Halo Objects) have been detected but they do not have sufficient density to explain flat rotation curves and cluster's velocities. Indeed, baryonic matter comprises 4% out of 26% of what is need to make matter in the universe [7]. We should also mention investigations which try to explain the flat rotation curves by resorting to non-Newtonian gravity [8].

In addition, recent observations of the redshift-luminosity relation observed in Type-Ia supernovae [9, 10, 11, 12, 13, 14, 15, 16] and the cosmic microwave background radiation (CMBR)[7] indicate that our universe is accelerating. The cosmic fluid is dominated by an unknown component with negative pressure which is called dark energy. The observations also suggest that the present universe is almost flat ( $\Omega_0 = 1.03 \pm 0.05$ ) with the energy density split into two main contributions:  $\Omega_{matter} \simeq 0.26$  (baryonic and dark), and  $\Omega_{D.E} \simeq 0.74$  (dark energy). Although the physical nature of dark energy is still a challenging problem, the cosmological constant is a first sight candidate. The cosmological constant energy density could originate from two sources. The first contribution is due to bare cosmological constant ( $\Lambda_0$ ), which appears in Einstein's equations and the second one is due to the vacuum energy density of quantum fields [18, 19, 20].

According to some observations, the dark energy density may be dynamical (vary slowly with time) in order to drive an accelerating universe. One possibility along this lines is the quintessence model, which suggests a scalar field  $\phi$  slowing rolling down a potential (a process like the inflationary model, in which a phase transition occurs due to a scalar field in the very early universe.) In this model, the scalar field  $\phi$  evolves very gradually with time, while being constant throughout the space.[see e.g. Peebles and Ratra 1998, Ratra and Peebles 1998, Wetterich 1998 Brax and Martin 1999,2000]. Quintessence is an interesting model that has attracted a great deal of attention (see also Refs. [26, 27, 28]).

On the other hand, some investigations have been carried out by invoking to a scalar field as the source of dark matter in spiral galaxies. The idea behind these works is to explore whether a scalar field can fluctuate along the history of the universe and thus form concentrations of scalar field energy density. Schunck (1998) has shown that an oscillating, massless, complex scalar field is capable of performing such a role [29]. More complicated systems with self-interaction potential, may also be considered for this purpose (see e.g. [30, 31, 32]).

In this paper, we would like to draw attention to the possibility of a unified picture to account for the existence of both dark matter and dark energy by a scalar field in current epoch, namely, after formation of objects such as galaxies.

We want to present a model in which the behavior of the  $\phi$ -field and in consequence  $V(\phi)$ , provides the corresponding energy density which could produce dark matter and dark energy inside and outside the halo, respectively. Along this lines, we will first consider a *massive complex* scalar field  $\Phi$  with a second-order potential of the form  $V(\Phi) \propto \Phi\Phi^*$ . For this scenario we consider a scalar field which is not only varying with

time (same as in quintessence model) but also is varying with position. Next, a *real* scalar field  $\phi$  undergoing spontaneous symmetry breaking will be considered. For this case, we assume that the real scalar field  $\phi$  is a function only of radial coordinate  $r$ .

## 2. Preliminaries

Einstein equations can be derived from the action

$$A = \frac{1}{2\kappa} \int (R + \mathcal{L}_{\text{matt}}(\Phi, \partial\Phi)) \sqrt{|g|} d^4x, \quad (1)$$

in which  $\mathcal{L}_{\text{matt}}$  is the Lagrangian density of matter,  $R$  is the curvature scalar,  $\kappa = 8\pi G$ ,  $g$  the determinant of the metric tensor  $g_{\mu\nu}$ , and  $\Phi$  is a complex scalar field. The variation of the action with respect to  $g_{\mu\nu}$  leads to Einstein's equations

$$G_{\nu}^{\mu} = R_{\nu}^{\mu} - \frac{1}{2} g_{\nu}^{\mu} R = -\kappa T_{\nu}^{\mu}. \quad (2)$$

The energy-momentum tensor in terms of the scalar field will be introduced shortly.

Variation with respect to  $\Phi$ , will lead to equation of motion for the scalar field in the background geometry

$$\square\Phi = 2 \frac{\partial}{\partial\Phi^*} V(\Phi, \Phi^*), \quad (3)$$

where  $\square$  is the invariant d'Alembertian defined by

$$\square = \frac{1}{\sqrt{|g|}} \partial_{\mu} [\sqrt{|g|} g^{\mu\nu} \partial_{\nu}]. \quad (4)$$

We first set up the metric for gravitational field within a spherically symmetric static dark halo. It reads

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2, \quad (5)$$

where  $A(r)$  and  $B(r)$  are arbitrary functions of the coordinate.

The non-vanishing components of the Einstein tensor for the metric (5) read

$$G_t^t = \frac{A'}{rA^2} + \frac{1}{r^2} - \frac{1}{r^2 A}, \quad (6)$$

$$G_r^r = \frac{-B'}{rAB} + \frac{1}{r^2} - \frac{1}{r^2 A}, \quad (7)$$

$$G_{\theta}^{\theta} = -\frac{2ABB' - 2A'B^2 + 2rABB'' - rAB'^2 - rA'B'B}{4rA^2B^2}, \quad (8)$$

$$G_{\phi}^{\phi} = G_{\theta}^{\theta}. \quad (9)$$

The energy-momentum tensor required to support such a spacetime is in the form

$$T_{\nu}^{\mu} = \text{diag}(-\rho, P_r, P_t, P_t), \quad (10)$$

where  $\rho$  is the energy density and  $P_r$  and  $P_t$  are the radial and transverse pressures, respectively.

Applying Einstein's equation (2) to the metric (5) gives

$$\rho(r, t) = \frac{1}{8\pi G} \left[ \frac{A'}{rA^2} + \frac{1}{r^2} - \frac{1}{r^2A} \right], \quad (11)$$

$$P_r(r, t) = -\frac{1}{8\pi G} \left[ \frac{-B'}{rAB} + \frac{1}{r^2} - \frac{1}{r^2A} \right], \quad (12)$$

$$P_t(r, t) = -\frac{1}{8\pi G} \left[ -\frac{2ABB' - 2A'B^2 + 2rABB'' - rAB'^2 - rA'B'B}{4rA^2B^2} \right], \quad (13)$$

Note that, in these equations dots and primes denote differentiation with respect  $r$  and  $t$ , respectively, and we have used the system of units in which  $c = 1$ .

From equations (11) and (12) we obtain

$$\frac{A'}{A} + \frac{B'}{B} = \kappa(\rho + P_r)rA, \quad (14)$$

and

$$\frac{A'r}{A^2} - \frac{1}{A^2} = \kappa\rho r^2 - 1. \quad (15)$$

The source is described by a complex scalar field with the Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi^* - V(\Phi, \Phi^*), \quad (16)$$

which leads to the following energy-momentum tensor

$$T_\nu^\mu = \partial^\mu\Phi\partial_\nu\Phi^* - \delta_\nu^\mu\left[\frac{1}{2}\partial_\lambda\Phi\partial^\lambda\Phi^* + V(\Phi, \Phi^*)\right]. \quad (17)$$

For the scalar field which evolves in the spherically symmetric background, we use the ansatz

$$\Phi(r, t) = \phi(r)e^{-i\omega t}, \quad (18)$$

where  $\omega$  is the frequency of the scalar field.

By using (5) and then (17) we obtain

$$\rho = -T_0^0 = \frac{1}{2B}\phi^2\omega^2 + \frac{1}{2A}\phi'^2 + V(\Phi, \Phi^*), \quad (19)$$

$$P_r = T_1^1 = \frac{1}{2B}\phi^2\omega^2 + \frac{1}{2A}\phi'^2 - V(\Phi, \Phi^*), \quad (20)$$

and

$$P_t = T_2^2 = T_3^3 = \frac{1}{2B}\phi^2\omega^2 - \frac{1}{2A}\phi'^2 - V(\Phi, \Phi^*). \quad (21)$$

Using (3) and (5), the differential equation governing the scalar field is obtained to be

$$\phi'' + \frac{1}{2}\left(\frac{B'}{B} - \frac{A'}{A} + \frac{4}{r}\right)\phi' + A\left(\frac{\omega^2}{B}\right)\phi(r) = 2Ae^{i\omega t}\frac{\partial}{\partial\Phi^*}V(\Phi, \Phi^*). \quad (22)$$

### 3. Solutions for the $\Phi^2$ - Potential

In this section, we perform some applications of the equations derived in the last section. The type of potential that is used to construct the model is a second-order polynomial in the complex scalar field  $\Phi$

$$V(\Phi) = m^2\Phi\Phi^* + V_0, \quad (23)$$

where  $m^2 > 0$  and  $V_0$  is a constant. We proceed with the Newtonian approach, since the gravitational field due to the dark and luminous matter is sufficiently weak [33].

By using the Minkowski metric in spherical coordinates, namely

$$ds^2 = -dt^2 + dr^2 + r^2d\theta^2 + r^2\sin^2\theta. \quad (24)$$

The scalar field equation (22) can be rewritten as

$$\phi'' + \frac{2}{r}\phi' + (\omega^2 - 2m^2)\phi(r) = 0, \quad (25)$$

and equations (19)-(21) take the forms

$$\rho = -T_0^0 = \frac{1}{2}\phi^2\omega^2 + \frac{1}{2}\phi'^2 + m^2\phi^2 + V_0, \quad (26)$$

$$P_r = T_1^1 = \frac{1}{2}\phi^2\omega^2 + \frac{1}{2}\phi'^2 - m^2\phi^2 - V_0, \quad (27)$$

$$P_t = T_2^2 = T_3^3 = \frac{1}{2}\phi^2\omega^2 - \frac{1}{2}\phi'^2 - m^2\phi^2 - V_0. \quad (28)$$

One can notice that, although the scalar field (18) is time dependent, the corresponding values of the non-vanishing components of the energy-momentum tensor are functions of the radial coordinate  $r$  only.

Let us define the following dimensionless quantities

$$\theta = \frac{\phi}{\phi_0}, \quad x = \frac{r}{r_0}. \quad (29)$$

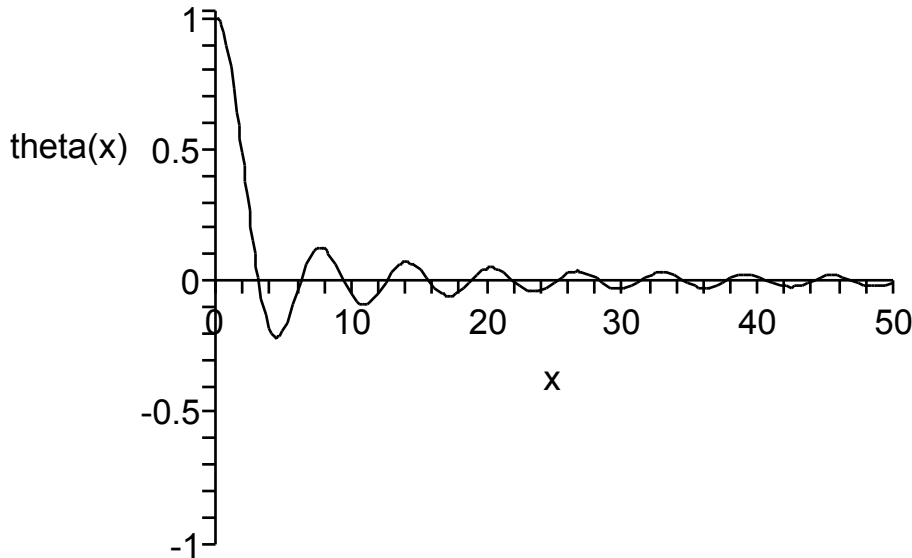
Then we can rewrite (25) and (26)-(28) as

$$\frac{d^2\theta}{dx^2} + \frac{2}{x}\frac{d\theta}{dx} + \alpha\theta = 0, \quad (30)$$

$$\rho = \gamma \left[ \beta\theta^2 + \left(\frac{d\theta}{dx}\right)^2 \right] + V_0, \quad (31)$$

$$P_r = \gamma \left[ \theta^2 + \left(\frac{d\theta}{dx}\right)^2 \right] - V_0, \quad (32)$$

$$P_t = \gamma \left[ \theta^2 - \left(\frac{d\theta}{dx}\right)^2 \right] - V_0, \quad (33)$$



**Fig. 1** The behavior of the dimensionless scalar field  $\theta = \frac{\phi}{\phi_0}$  as a function of the dimensionless coordinate  $x = \frac{r}{r_0}$ .

in which  $\alpha = r_0^2(\omega^2 - 2m^2)$ ,  $\beta = \frac{\omega^2 + 2m^2}{\omega^2 - 2m^2} \neq 1$ ,  $\gamma = \frac{1}{2} \frac{\varphi_0^2}{r_0^2}$  are dimensionless parameters. For  $\alpha = \gamma = 1$ , equation (30) has the solution

$$\theta(x) = \frac{1}{x} [C \sin x + D \cos x], \quad (34)$$

where  $C$  and  $D$  are constants. The solutions that we are interested in are required to be non-singular at the origin, so we proceed with  $D = 0$ . The function  $\theta(x)$  is plotted in figure 1.

From the solution (34), equations (31)- (33) can be rewritten in the following forms

$$\rho = \frac{C^2}{x^2} \left[ \beta \sin^2 x + \cos^2 x - \frac{\sin 2x}{x} + \frac{\sin^2 x}{x^2} \right] + V_0, \quad (35)$$

$$P_r = \frac{C^2}{x^2} \left[ 1 - \frac{\sin 2x}{x} + \frac{\sin^2 x}{x^2} \right] - V_0, \quad (36)$$

$$P_t = \frac{C^2}{x^2} \left[ -\cos 2x + \frac{\sin 2x}{x} - \frac{\sin^2 x}{x^2} \right] - V_0. \quad (37)$$

The energy density of the  $\phi$ -field depends sensitively on the value of the dimensionless quantity  $x$ . Inside the halo, the contribution of the energy density due to the first four

terms -corresponding to dark matter -prevails. By increasing the distance from the center of the halo, these terms diminish and the  $\phi$ -field goes to the vacuum state of the potential and its energy density reduces to the constant value  $V_0$ . Accordingly, outside the halo, the components of the stress-energy tensor will have the following forms

$$\rho = V_0, \quad (38)$$

$$P_r = P_t = -V_0, \quad (39)$$

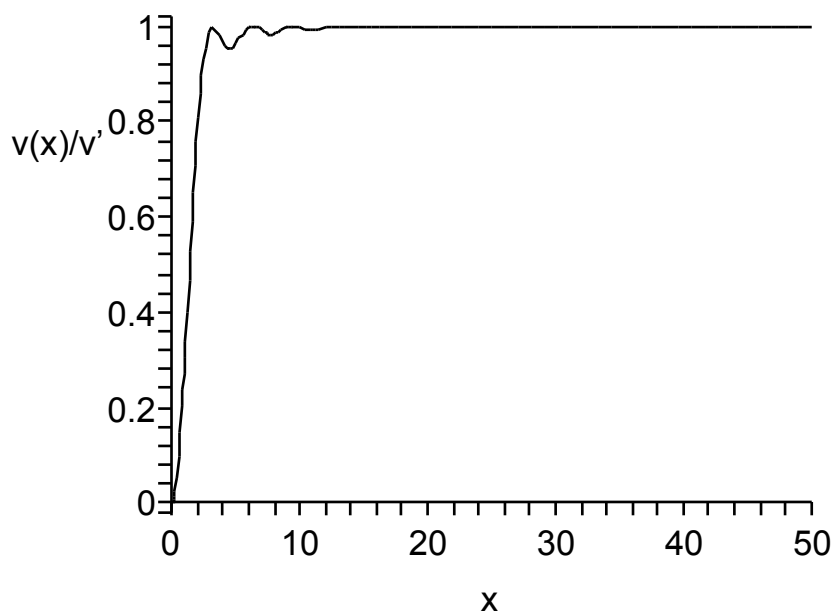
and

$$P_r = P_t = -\rho. \quad (40)$$

This is the familiar form of the equation of state for the cosmological constant, which is grossly considered as the origin of the dark energy in the present universe.

This implies that

$$V_0 = \frac{\Lambda}{8\pi G}. \quad (41)$$



**Fig. 2** A generic flat rotation curve  $v_{rot}/v'$ , for  $\beta = 10$ .

By using the mass function  $m(x) = \int_0^x \rho(\xi)\xi^2 d\xi$ , the Newtonian mass of the dark matter is given by

$$M(x) = M' \left[ \beta \left( \frac{x}{2} - \frac{1}{4} \sin 2x \right) + \left( \frac{x}{2} + \frac{1}{4} \sin 2x \right) + \frac{\cos 2x - 1}{2x} \right], \quad (42)$$

where  $M' = 4\pi r_0^3 C^2$ .

For a weak gravitational field, we have the Newtonian formula

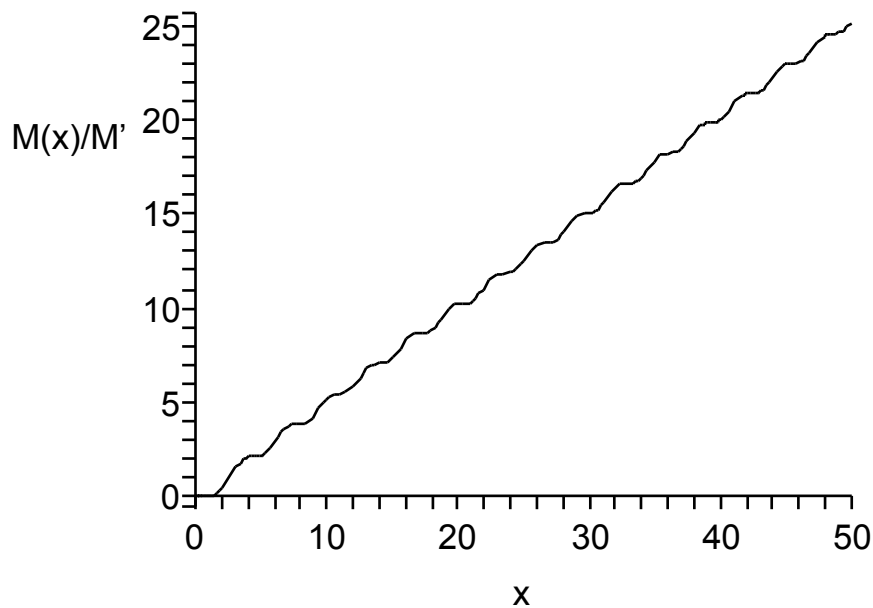
$$v^2 = G \frac{M(r)}{r}. \quad (43)$$

Substituting (42) into this equation, yields

$$v_{rot}^2 = v'^2 \left[ \frac{\beta}{x} \left( \frac{1}{2}x - \frac{1}{4} \sin 2x \right) + \frac{1}{x} \left( \frac{1}{2}x + \frac{1}{4} \sin 2x \right) - \frac{\sin^2 x}{x^2} \right]. \quad (44)$$

The corresponding plot (for the tentative value  $\beta = 10$ ) is presented in figure 2. Note that the coefficient  $v'^2 = 4\pi G r_0^2 C^2$ , which appears in equation (44), determines the halo characteristic mass. Subsequently, depending on the value of this coefficient, we can fit appropriate rotation curves for different galaxies.

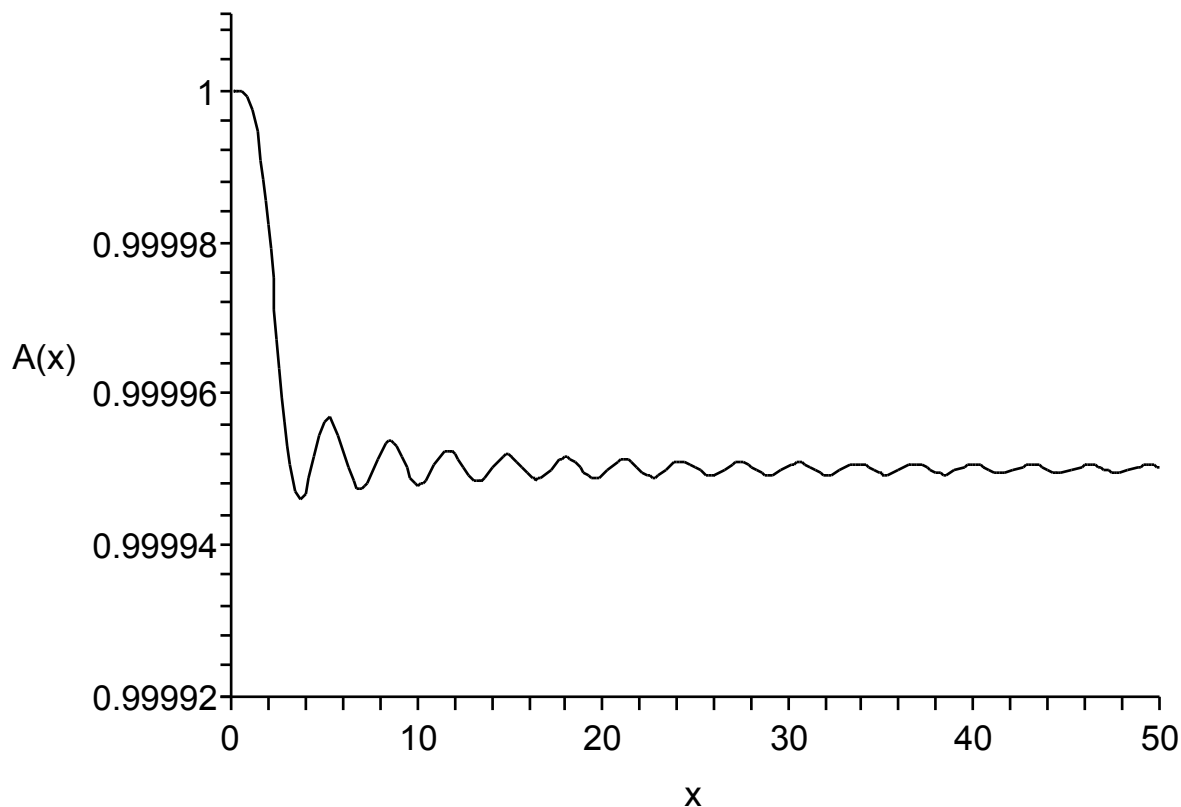
For  $\beta = 10$ , the typical behavior of  $M(x)/M'$  is shown in figure 3.



**Fig. 3** The Newtonian mass function  $M(x)/M'$ , for  $\beta = 10$ .

### 3.1 GR Solutions

From viewpoint of astrophysics, gravitational fields are often very weak, but sometimes the correction effects of general relativity become important (e.g. the precession of Mercury). Let us consider the general-relativistic effects on rotation curves of galaxies.



**Fig. 4** The metric function  $A(x)$ .

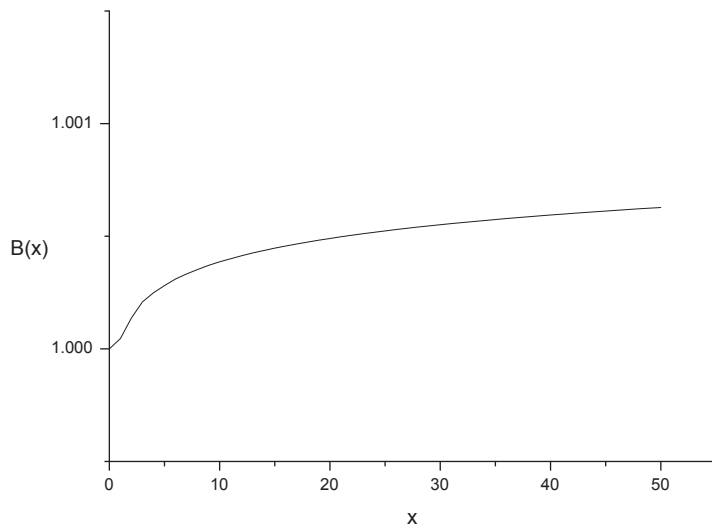
The nearly constant rotational velocity of spiral galaxies shows that the amount of dark matter grows continuously with increasing the distance from the center of the halo. For this configuration, the values of  $A(r)$  and  $B(r)$  are [35]

$$A(r) = \left[ 1 - \frac{2GM(r)}{r} \right]^{-1}, \tag{45}$$

$$B(r) = B(r_1) \exp \left\{ - \int_r^{r_1} \frac{2G}{r'^2} [M(r') + 4\pi r'^3 P_r] A(r') dr' \right\}. \tag{46}$$

In order to obtain the approximate value of these functions, we use the values of mass and radial pressure from the Newtonian solutions of the previous section, namely, equations (36) and (42). By inserting these equations into (45) and (46), we obtain

$$A(x) \approx 1 - 2v'^2 \left[ \frac{\beta}{x} \left( \frac{x}{2} - \frac{1}{4} \sin 2x \right) + \frac{1}{x} \left( \frac{x}{2} + \frac{1}{4} \sin 2x \right) + \frac{\cos 2x - 1}{2x^2} \right]^{-1}, \tag{47}$$



**Fig. 5** The metric function  $B(x)$ .

and

$$B(x) = B(x_1) \exp \int_x^{x_1} -\frac{2v'^2}{x^2} \left[ \beta \left( \frac{x}{2} - \frac{1}{4} \sin 2x \right) + \left( \frac{x}{2} + \frac{1}{4} \sin 2x + \frac{\cos 2x - 1}{2x} \right) + x - \sin 2x + \frac{\sin^2 x}{x} \right] \left[ 1 - 2v'^2 \left[ \frac{\beta}{x} \left( \frac{x}{2} - \frac{1}{4} \sin 2x \right) + \frac{1}{x} \left( \frac{x}{2} + \frac{1}{4} \sin 2x \right) + \frac{\cos 2x - 1}{2x^2} \right] \right]^{-1}. \quad (48)$$

note that we have used the dimensionless variable  $x = \frac{r}{r_0}$ . Figure 4 shows the behavior of  $A(x)$  as function of  $x$ . In order to determine  $B(x)$ , a numerical calculation was carried out. The result is illustrated in figure 5.

From equation (22), we have

$$\phi'' + \frac{1}{2} \left( \frac{B'}{B} - \frac{A'}{A} + \frac{4}{r} \right) \phi' + A \left( \frac{\omega^2}{B} - 2m^2 \right) \phi(r) = 0. \quad (49)$$

Let us simplify this equation. By inserting (6), (7) and (11), (12) into Einstein equation (2), we have

$$\frac{B'}{rAB} - \frac{1}{r^2} + \frac{1}{r^2 A} = \kappa \left[ \frac{1}{2A} \phi'^2 + \frac{1}{2B} \omega^2 \phi^2 - m^2 \phi^2 \right], \quad (50)$$

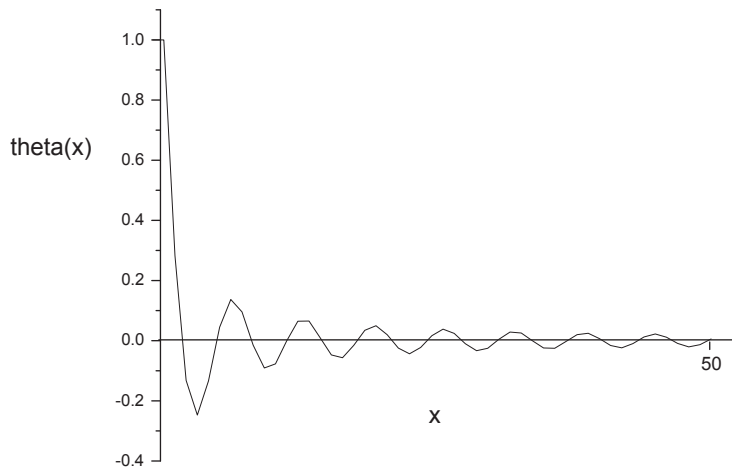
$$\frac{A'}{rA^2} + \frac{1}{r^2} - \frac{1}{r^2 A} = \kappa \left[ \frac{1}{2A} \phi'^2 + \frac{1}{2B} \omega^2 \phi^2 + m^2 \phi^2 \right]. \quad (51)$$

Subtracting 6 from (51), we obtain

$$\frac{B'}{B} - \frac{A'}{A} = \frac{2A}{r} - \frac{2}{r} - 2\kappa r A m^2 \phi^2, \quad (52)$$

substituting this equation into (49), yields

$$\frac{d^2 \phi}{dr^2} + \frac{1}{r} (A + 1) \frac{d\phi}{dr} - \kappa r A m^2 \phi^2 \phi' + A \left( \frac{\omega^2}{B} - 2m^2 \right) \phi = 0. \quad (53)$$



**Fig. 6** The general relativistic behavior of  $\theta(x)$  for  $\xi = 1$ .

From the dimensionless quantities (29), the preceding equation can be written in the alternative form

$$\frac{d^2\theta}{dx^2} + (A + 1)\frac{1}{x}\frac{d\theta}{dx} - \left(\frac{\beta - 1}{2}\right) \left[ -\xi x A \theta^2 \frac{d\theta}{dx} + \left(\frac{A}{B} - A\right)\theta \right] + \frac{A}{B}\theta = 0, \quad (54)$$

in which  $\xi = \kappa r_0^2$  is a dimensionless parameter. As boundary condition, we apply  $\theta = 0$ ,  $\frac{d\theta}{dx} = 0$ , at large enough  $x$ , the numerical calculation determines the values of  $\theta$  at each point. The result is illustrated in figure 6

For the static, spherically symmetric metric (5) the tangential velocity of a test particle obeys

$$v_{rot}^2 = \frac{1}{2}rB'. \quad (55)$$

From the dimensionless quantities (29), we have

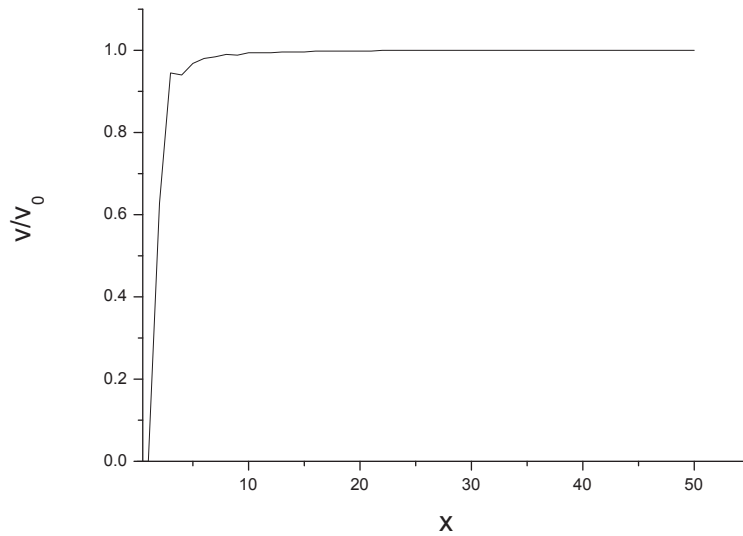
$$v_{rot}^2 = \frac{1}{2}x\frac{dB}{dx}, \quad (56)$$

we have calculated  $v_{rot}/v_0$  numerically. The outcome can be seen in figure 7. Note that, in this case  $v_0$  is the value of  $v_{rot}$  at large  $x$ . As expected, figures 6 and 7 show that the deviation from the Newtonian solution is not considerable.

#### 4. Solutions for the $\phi^4$ - Potential

The flat rotation curve results of the  $\Phi^2$ -potential, seem to be promising, but it is instructive to use a similar approach for another celebrated potential in the field theory, known as the  $\phi^4$ -potential:

$$V(\phi) = \frac{1}{2}\lambda(\phi^2 - \phi_0^2)^2, \quad (57)$$



**Fig. 7** General relativistic rotation curve as a function of the dimensionless distance from the center of dark halo.

where  $\lambda > 0$ . In this case, we assume that the real scalar field  $\phi$  is a function only of  $r$ . As illustrated in figure 8, the potential has one local maximum (false vacuum) at  $\phi = 0$  and two global minima (true vacuums) at  $\pm\phi_0$ .

As pointed out before, the Newtonian limit is adequate for our investigation. So, we proceed in this framework.

For the real massless scalar field  $\phi(r)$ , the source is described with Lagrangian

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi). \quad (58)$$

The Euler-Lagrange equation leads to the differential equation for the scalar field

$$\square\phi = \frac{dV(\phi)}{d\phi}. \quad (59)$$

For  $\phi$  a function of  $r$  only, we have

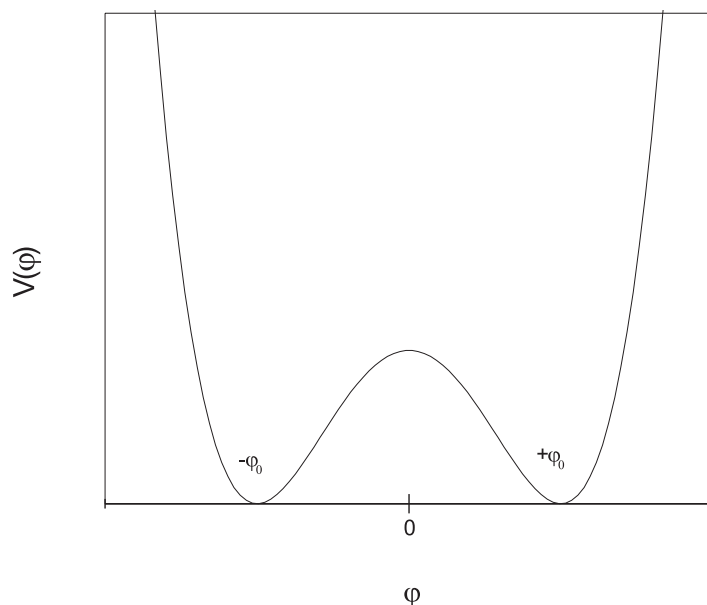
$$\frac{d^2\phi(r)}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{dV(\phi)}{d\phi}. \quad (60)$$

Inserting (57) into this equation, we get

$$\frac{d^2\phi(r)}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} - 2\lambda(\phi^2 - \phi_0^2)\phi = 0. \quad (61)$$

The stress tensor  $T_\nu^\mu$  related to Lagrangian (58) is

$$T_\nu^\mu = \partial^\mu\phi\partial_\nu\phi - \delta_\nu^\mu \left[ \frac{1}{2}\partial_\lambda\phi\partial^\lambda\phi + \frac{1}{2}\lambda(\phi^2 - \phi_0^2)^2 \right]. \quad (62)$$



**Fig. 8** Spontaneous symmetry breaking potential for the scalar field (the so-called  $\phi^4$ -potential)

The non-vanishing components of the energy momentum tensor are

$$\rho = -T_0^0 = \frac{1}{2}\phi'^2 + \frac{1}{2}\lambda(\phi^2 - \phi_0^2)^2, \quad (63)$$

$$P_r = T_1^1 = \frac{1}{2}\phi'^2 - \frac{1}{2}\lambda(\phi^2 - \phi_0^2)^2, \quad (64)$$

and

$$P_t = T_2^2 = T_3^3 = -\frac{1}{2}\phi'^2 - \frac{1}{2}\lambda(\phi^2 - \phi_0^2)^2, \quad (65)$$

in which we have used the metric (24) for flat space. By using the dimensionless quantities (29), equations (61) and (63) can be written as

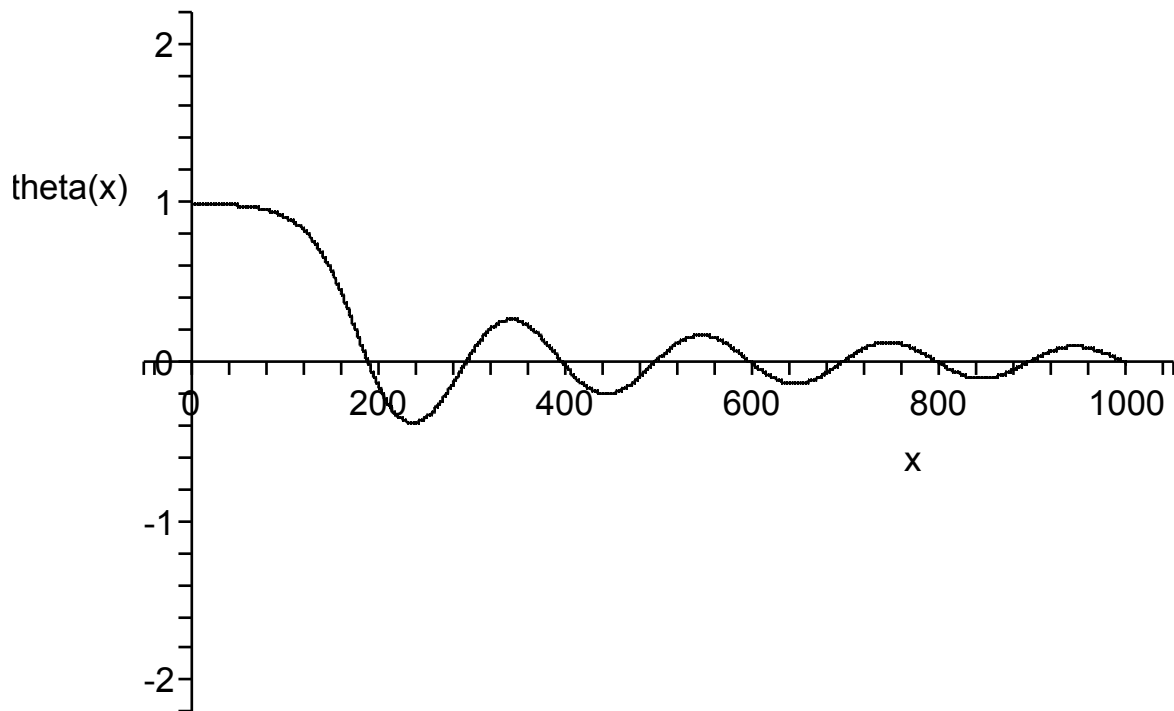
$$\frac{d^2\theta}{dx^2} + \frac{2}{x} \frac{d\theta}{dx} - \alpha'(\theta^2 - 1)\theta = 0, \quad (66)$$

and

$$\rho = \beta' \left( \frac{d\theta}{dx} \right)^2 + \gamma'(\theta^2 - 1)^2, \quad (67)$$

where  $\alpha' = 2\lambda r_0^2 \phi_0^2$ ,  $\beta' = \frac{1}{2} \frac{\phi_0^2}{r_0^2}$  and  $\gamma' = \frac{1}{2} \lambda \phi_0^4$  are dimensionless parameters.

In order to get regular solutions for the differential equation (66), we put the initial conditions  $\theta(0) = 1$  and  $\frac{d\theta}{dx}(0) = 0$ . By assuming  $\alpha' = 0.001$  the numerical calculation was carried out and the function of  $\theta(x)$  is illustrated in figure 9. As it is seen, the scalar



**Fig. 9** Generic behavior of the function  $\theta(x)$  for the  $\phi^4$ -model ( $\alpha' = 0.001$ ,  $\theta(0) = 1$  and  $\frac{d\theta(0)}{dx} = 0$ ).

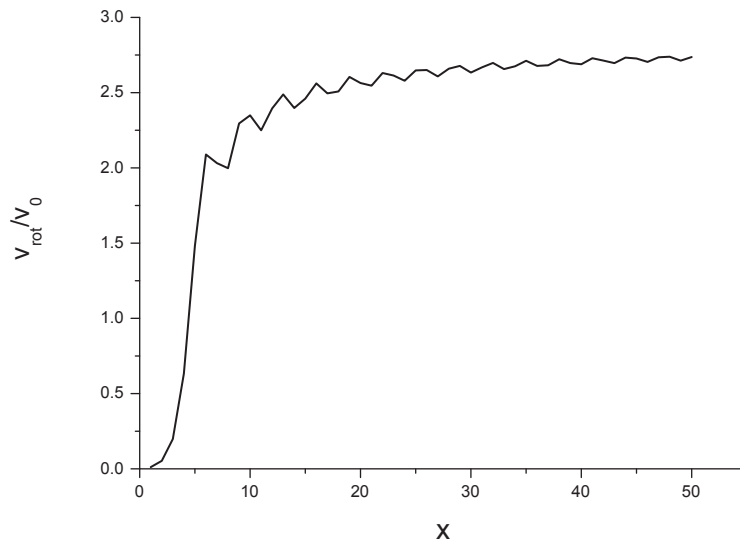
field is nearly at the true vacuum of the potential at origin. However, for large values of  $x$ , the  $\phi$ -field fluctuates around its false vacuum.

Let us investigate some properties of the corresponding energy density  $\rho$  which makes it a particular and interesting case, by taking into account equation (67) and the behavior of  $\theta(x)$  which is plotted in figure 9. When the scalar field is near to the true vacuum of the potential ( $\theta = 1$ ), by a suitable choice of  $\beta'$ , the first term of equation (67) prevails the second one, and we interpret this region as inside the halo and the corresponding value of the energy density is regarded as the dark matter energy density. However, for large values of  $x$ , the first term can be ignored and this corresponds to outside the halo, where the scalar field goes to zero and the energy density (67), approaches the constant value

$$\rho \rightarrow V(0) = \frac{1}{2}\lambda\phi_0^4. \quad (68)$$

In such a case, equations (64) and (65) become

$$P_r = P_t = -V(0), \quad (69)$$



**Fig. 10** Rotation curve of the  $\phi^4$ -model for  $\alpha' = 0.001$ ,  $\theta(0) = 1$  and  $\frac{d\theta(0)}{dx} = 0$ , in which  $v_0^2 = 4\pi Gr_0^2 \rho_\Lambda$ .

and  $\rho = -P$ , which is the equation of state of the cosmological constant. So we can write

$$V(\phi = 0) = \frac{1}{2} \lambda \phi_0^4 = \rho_\Lambda, \tag{70}$$

in which  $\rho_\Lambda \simeq 10^{-9} \frac{\text{joul}}{\text{m}^3}$ . This equation determines the value of  $\lambda$  :

$$\lambda = \frac{2\rho_\Lambda}{\phi_0^4}. \tag{71}$$

From the preceding equation, together with the value of  $\alpha' = 2\lambda r_0^2 \phi_0^2$ , we can easily obtain

$$\beta' = \frac{2\rho_\Lambda}{\alpha'}. \tag{72}$$

So, equation (67) can be rewritten as

$$\rho = \rho_\Lambda \left[ \frac{2}{\alpha'} \left( \frac{d\theta}{dx} \right)^2 + (\theta^2 - 1)^2 \right]. \tag{73}$$

The first term of this equation (which corresponds to the dark matter energy density) enables us to calculate rotation curve from the Newtonian formula (43). It has been obtained using numerical calculations and the result is shown in figure 10. It should be emphasized that the dimensionless parameter  $\alpha'$  which appears in (73) determines the halo characteristics mass and density near the center. In other words, by suitable choice of  $\alpha'$  we can get favorable rotation curves for different galaxies.

From (73), it is clear that for inside halo, the first term (energy density of dark matter) should be much greater than the second term (energy density of dark energy). For this purpose, we need  $\alpha' < 1$ .

## Conclusion

From the rotation curves of spiral galaxies and velocities of galaxies in clusters, the existence of dark matter has long been confirmed by astronomers.

On the other hand, reliable observational evidence from WMAP and Type-Ia supernovae results, not only confirm the existence of dark matter but also indicate that the present universe is almost flat, and consists of 4% baryons, 22% dark matter and 74% dark energy.

We presented a model, in which the energy density of a spherically symmetric scalar field could produce both the dark matter and dark energy inside and outside of the halo, respectively.

We started from a general, spherically symmetric line element. The  $\Phi$ -field was described by  $\mathcal{L} = \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi^* - V(\Phi, \Phi^*)$ , so that the corresponding stress tensor  $T_\nu^\mu$  has form  $T_\nu^\mu = \text{diag}(-\rho, P_r, P_t, P_t)$ .

As a special case, we considered a complex scalar field  $\Phi$  with a second-order potential of the form  $V(\Phi) \propto \Phi\Phi^*$ , in which  $\Phi(r, t) = \phi(r)e^{-i\omega t}$ . In the Newtonian framework, we found the exact solution for  $\phi(r)$  and, consequently the corresponding energy density. The corresponding energy density was found to be high at the center and reduce to the constant value  $V_0$  at large radii. Accordingly, the scalar field fluctuation could serve as dark matter and dark energy inside and outside the halo, respectively. By using these solutions, rotational velocity was computed and the corresponding rotation curve was obtained. By suitable choices for  $v'$  (halo characteristic parameter), we can fit appropriate rotation curves for different galaxies. The approximate GR solutions were also obtained, although the deviation from the Newtonian results were found to be quite small.

Furthermore, we presented a model with a real scalar field  $\phi$  as a function only of coordinate  $r$ . We employed the  $\phi^4$ -potential which is known to cause spontaneous symmetry breaking. We demonstrated how the scalar field can, in principle, reproduce dark matter energy density for inside and dark energy for outside the halo. Inside the halo, the rotational velocity was computed numerically and, shown to exhibit an almost flat behavior.

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## References

- [1] F. Zwicky, *Helv. Phys. Acta.* **6**, 110 (1933).
- [2] V.C. Rubin and W.K. Ford *Astrophys. J.* **159**, 379 (1970).
- [3] M.S. Roberts and A.H. Rots, *Astr. Astrophys.* **26**, 483 (1973).
- [4] D.H. Rogstad and G.S. Shostak, *Astrophys. J.* **176**, 315 (1972).

- [5] N.P. Vogt, D.A. Forbes, A.C. Phillips, C. Gronwall, S.M. Faber, G. D. Illingworth, and D.C. Koo *Astrophys. J.* **465**, L15 (1996).
- [6] T. Brainerd, R. Blanford, and I. Smail, *Astrophys. J.* **466**, 623 (1996).
- [7] D.N. Spergel *Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Results: Implications for Cosmology*, [astro-ph/0603449].
- [8] M. Milgrom, *Astrophys. J.* **333**, 689 (1988).
- [9] P.M. Garnavich *et al.* [Hi-Z Supernova Team Collaboration], *Astrophys. J.*, **493**, L53, (1998)[astro-ph/9710123].
- [10] B.P. Schmidt *et al.*, *Astrophys. J.*, **507**, 46, (1998)[astro-ph/9805200].
- [11] A.G. Riess *et al.*, *Astrophys. J.*, **116**, 1009, (1998)[astro-ph/9805201].
- [12] P.M. Garnavich *et al.*, *Astrophys. J.*, **509**, 74, (1998)[astro-ph/9806396].
- [13] S. Perlmutter *et al.*, *Astrophys. J.*, **483**, 565, (1997)[astro-ph/9608192].
- [14] S. Perlmutter *et al.*, *Nature*, **391**, 51, (1998)[astro-ph/9712212].
- [15] S. Perlmutter *et al.*, *Astrophys. J.*, **517**, 565, (1999)[astro-ph/9612133].
- [16] R.A. Knop *et al.*, arXiv:astro-ph/0309368.
- [17] D.N. Spergel *et al.*, *Astrophys. J. Suppl.*, **148**, 175, (2003)[astro-ph/0302209].
- [18] S.M. Carrol, *Introduction to Cosmology*, [arxiv.astro-phy/0400154].
- [19] S.M. Carrol, *Why is Universe Accelerating*, [arxiv.astro-phy/0310342].
- [20] T. Padmanabhan, *Cosmological Constant-The Weight Of The Vacuum*, [arxiv.astro-phy/0212290].
- [21] B. Ratra and P.J.E Peebles, *Phys. Rev. D*, **37**, 3406 (1988).
- [22] P.J.E Peebles and B. Ratra, *Ap. J. Lett.* **325**, L17 (1988).
- [23] C. Wetterich, *Nucl. Phys. B* **302**, 645 (1988).
- [24] B. Ratra and P.J.E Peebles, *Phys. Rev. D*, **37**, **3406** (1988).
- [25] P. Brax, and J. Martin, *Phys. Lett B*, **468**, 40 (1999)[astro-ph/9905040].
- [26] T. Barreiro and E.J. Copeland and N.J. Nunes, *Phys. Rev.* **D61** (2000)[astro-ph/9910214].
- [27] V. Sahni and L. Wang, (1988) [astro-ph/9910097].
- [28] I. Zlatev, L. Wang and P.J. Steinhardt, *Phys. Rev. Lett.* **82**, 896 (1988)[astro-ph/9807002].
- [29] F.E. Schunck, (1998) [astro-ph/9802258v1].
- [30] A. Matos, F. Guzman, and D. Nunez, *Phys. Rev.*, **D62**, 061301, (2004)[astro-ph/9910214].
- [31] A. Arbey, *Phys.Rev.D*, **74**, 043516, (2006) [astro-ph/0601274].
- [32] A. Arbey, *EAS Publ.Ser.*, **20**, 257-260, (2006)[astro-ph/0509592].
- [33] V. Sahni, (2004)[astro-ph/0403324].
- [34] S. Weinberg, *Gravitation and Cosmology*, John Wiley and Sons, USA, (1972).
- [35] S. Carrol, *Living Rev. Rel.*, **4**, 1, (2001)[astro-ph/0004075].

